Chevigne

1er ed., 1806

MATHEMATICAL MANUAL

FOR THE USE OF

COLLEGES AND ACADEMIES.

VOLUME FIRST.

CONTAINING FOUR PARTS;

—VIZ :—

I. *RATIONAL ARITHMETIC,*

II. *ELEMENTS OF ALGEBRA,*

III. *PRACTICAL ARITHMETIC,*

IV. *PRACTICAL ALGEBRA.*

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DISTRICT OF MARYLAND ;—TO WIT.

BE IT REMEMBERED, That on the Twenty-third day of March, and in the Thirty-first year of the Independence of the United States of America, L. I. M. Chevigne, of the said District, hath deposited in this office the title of a Book, the right whereof he claims as Proprietor, in the words and figures following :--*to wit*:-- “ Mathematical Manuel, for the use of Colleges and “ Academies, Volume First; containing four Parts, *viz.* “ [illegible]ional Arithmetic, II. Elements of Algebra, “ III. Practical Arithmetic, IV. Practical Algebra.” In conformity to the Act of the Congress of the United States, entitled, “ An Act for the encouragement of learning, by securing the Copies of Maps, Charts, and Books, to the Authors and Proprietors of such Copies, during the times therein mentioned.”

PHILIP MOORE,

Clk. Dist. of Maryland.

INTRODUCTION.

THE object of the two firſt parts has been to preſent to the young learner, in the moſt conciſe terms, the reaſons of the various operations in Arithmetic and Algebra.

The principal ſcope of a college education being to unfold and expand the intellectual faculties of youth, and no ſtudy being better calculated for that purpoſe than that of the Mathematics, it would be a great miſtake to offer to the mere exertions of memory, a ſcience chiefly intended to exerciſe and cultivate the underſtanding. Experience, beſides, has ſhewn, that a ſyſtem of *routine* requires a much longer time, and is attended with much greater labor, both for the teacher and the pupil, than that which is wholly founded upon reaſon.

We may add, that a youth thus taught, as it were, mechanically, will ſoon loſe all the fruits of his long and painful efforts, unleſs he is in the way of a continual practice, and (a miſchief of ſtill greater magnitude) that ſuch a method tends to damp that activity of the mind the nobleſt prerogative of the human ſpecies above the brute creation.

With an intention, therefore, of giving memory as little as possible to do, the two firſt parts exhibit only *general rules*, often eaſier to be underſtood than *particular methods*, and which being few in number, are of courſe more eaſily retained, and at once preſent to view, in a ſmall compaſs, the whole ſyſtem of the ſcience.

The firft part contains a new method for the rule of three. The general manner in which all queftions on this rule are confidered, fuperfedes the diftinction of *direct* and *inverfe*; a fource of difficulty for beginners, and of confufion for practitioners. Experience feems to have proved in favor of this method.

Algebra has been reduced to fuch queftions as are fufficient to underftand *application of Algebra to geometry*, *conic fections*, *fluxions*, and *the application of the fluxionary calculation* to fome ufeful queftions in *mechanicks* and *hydro-dynamicks*.

No general method has been given for folving the equations of the third and fourth degrees; becaufe the procefs is long and laborious, and leads to general formulæ extremely complicated, which are truly interefting to none but profound algebraifts; fince, in practice, all particular cafes can be folved by the way of trial, as it has been explained.

The third part is a felection from the books generally ufed in fchools, of the queftions and examples which have appeared moft interefting All the methods of contraction in ufe have been given, with an explication of their theory, in order that the learner may fully underftand the reafons of all the operations they are made to perform. Such pupils as may be deftined to fituations in life, which require a great practice of calculation, will find in this third part, all the methods and examples conducive to that purpofe.

The fourth part contains feveral rules and problems of Algebra.

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CONTENTS.

ERRATA.

Page	Line	FAULTS.	READ
24	4	left	right.
29	32	$\frac{60\times 8+3}{12}$	$\frac{60+8+3}{12}$
31	23	$4+3+3+5$	$4\times 3\times 3\times 5$
32	15	$\frac{72-96+100-36}{144}=\frac{172-132}{144}=\frac{40}{44}$	$\frac{72-96+120-36}{144}=\frac{192-132}{144}=\frac{60}{184}$
34	22	$2+5$	2×5
38	13	663×221	$663+221$
38	18	2431×884	$2431+884$
38	30	2431×884	$2431+884$
40	33	and making	in making.
41	11	9*d*	0*d*.
45	11	8	3

ERRATA.

Page	Line	Faults.	Read
59	23	12 ·· 4 :: 14 ·· 6	12 : 4 :: 14 : 6
105	...	$-3b^2$ (in the Divisor)	$-3b^3$
119	23	$(x+a^{\frac{1}{2}}$	$(x+a)^{\frac{1}{2}}$
128	7	b^2d } repeated all along the calculation.	b^2
136	22	with the second	with the third.
138	...	360 dollars (problem)...	360d.
139	12	the first condition	the first equation.
142	31	[illegible]	[illegible]
152	13	sum.	product.
159	27	$S=(2a-d\,\overline{n-1}\,\frac{n}{2})$ ...	$S=(2a-d\,.\,\overline{n-1})\,\frac{n}{2}$
161	20	proportion....................	progression.
162	9	by Example.................	for Example.
166	22	pass by such...............	pass through.
170	8	2,2[illegible]2441............	2,262451
173	30	2654800.	2655214
174	9	[illegible] (& in two other places)	$\frac{1}{n}$
174	20	0,026109	See the note, page 275.
182	5	*Ans.* 1004	*Ans* 1104
184	11	Product 884,630.	885,1030
188	27	(Quotient) 207 + $\frac{2}{34}$	230 + $\frac{20}{34}$
208	10	*Ans* 1811*lb* 3*oz.* 13*dwt.* $\frac{7}{8}$	1811*lb* · 1*oz* · 13*dwt.* [illegible]
219	...	$\frac{1}{8}d.$ (4th Example)	$\frac{1}{3}d.$
241	10	*Ans*.........$\frac{1}{2}d$...............	$\frac{3}{4}d$
242	14	*Ans* ....$3\frac{3}{4}d$	$4\frac{1}{2}d$
245	...	cube root of 6983587...	65450827

ARITHMETIC, &c.

GENERAL PRINCIPLES OF

NUMERATION.

ARITHMETIC is the SCIENCE OF NUMBERS. Numbers are expressed by means of ten characters called figures; *viz.* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The manner of using these ten figures, to express every number imaginable, is called the *Art of Numeration*, which is founded upon this principle, that, if several figures be written in an horizontal line, such as 5473, the units of any one of these figures, of 7 for instance, are each of them worth ten times a unit of the figure placed at its right, and ten times less than the units of the figure at its left. Thus, the first fig. 3 expressing three single units, the fig. 7 will express seven tens of the same units; the fig. 4, four tens of tens or four hundred, and so on; ten units of any fig. being equal to one unit of the fig. placed immediately at its left.

This principle being once settled, another convention has been necessary, for reading easily every sort of numbers. Let us suppose a considerable number,

billions. millions. thousands. units.

such as 4 | 578 | 632 | 459; this number is *mentally* divided into members of three figures each, begin-

ning by the right, and proceeding towards the left; each of which members having its peculiar appellation; namely, the first on the right, being called the member of *units*; the second, of *thousands*; the third, of *millions*; the fourth, of *billions*, &c. &c.——The members being thus denominated, can be each of them read separately, considering that each complete member contains *units*, *tens*, and *hundreds*. Let us, for instance, take the member of *thousands*: If the first fig. 2, represents *thousands*, the 3, at its left, will express *tens of thousands*; and the 6, at the left of the 3, *hundreds of thousands*; so that this member contains Six hundred thirty two *thousands*. On this principle, the total number will be thus read,—*four billions, five hundred and seventy eight millions, six hundred and thirty two thousand, four hundred and fifty nine units.*

Whereupon it is to be observed,—Firstly, that the 0, or cypher, which by itself has no value, answers nevertheless two important objects in a number. It not only indicates that there are no units of the order in which it is placed in that number, but also serves to give a tenfold value to the figures placed at its left. Thus, for instance, in the number 3704, the cypher shews, in the first place, that there are no *tens*; then it is evident, that it makes all the figures, at its left, ten times greater than they would have been, had the cypher been omitted; for, if we take it off, and write 374, the 7, which expressed *hundreds*, would now indicate only *tens*; and the 3, which was of *thousands*, would now only be *hundreds*:

Therefore, Secondly; that in order to make a number 10, 100, 1000, &c. times greater, it suffices to write one, two, or three, &c. cyphers at the right of that number. Let us take the number 25; if I add a cypher and write 250, the fig. 5, which was of *units*, becomes *tens*; and the 2, which expressed *tens*,

now indicates *hundreds* ; consequently each of these two figures has become ten times greater; therefore the whole number has been itself made ten times greater. ——By a similar reasoning, we shew, that if we put two cyphers, and write 2500, the number 25 has been made one hundred times greater, and so on.— It is sufficiently evident, that cyphers, placed at the left of a number, alter not its value.

Thirdly ; that, if we suppress one, two, three, &c. cyphers at the right of a number, we make it 10, 100, 1000, &c. times smaller ; this is a necessary consequence of the foregoing remark.

Conformably to the above principles, if we wish to express the number *ten*, we write 10, since 1 being, by the accession of 0, made ten times greater, becomes a *ten*. To express *eleven*, we write 11, that is *one ten*, more *one unit* ; for *twelve*, we should write 12, which is *one ten* and *two units* ;—to express *twenty five*, we write 25, or *two tens* and *five units*. Proceeding on the same way, we see that it is easy to form the series of *natural numbers*, which will be infinite, since we may always add a figure to any number, however great it may be supposed.

OF DECIMAL PARTS.

If we conceive the *unit* divided into *ten* equal parts, then each of these into other *ten*, and so on, we shall have the notion of a series of new numbers, expressive of parts of the principal unit : these are what we called *Decimals*. But in the same way, in which, by progressively *decupling* the principal unit, we have formed *tens*; *hundreds*, *thousands*, &c. which compose an ascending *suite* from the right to the left, we may take a descending one, in the opposite order, which progressively decreases in a tenfold proportion, such

as the *tenth*, *hundredth*, *thousandth*, &c. part of the principal unit.

To distinguish the decimal parts from the principal units, we separate the ones from the others by a *comma*, thus the number 53,4, expresses fifty three units, and four tenths of a *unit* ; if we put another fig. at the right of the 4, such as 53,47, each unit of the 7, will express *tenths of tenths*, or *hundredths*.—If we had the number 53,476, the 6 would express *tenths of hundredths*, or *thousandths* ; a fourth fig. would be *tenths of thousandths* ; a fifth, *hundredths of thousandths*, and so on infinitely. We may then call this last number in the following manner ; *fifty three units*, *four tenths*, *seven hundredths*, *six thousandths*.—But since a *hundredth* is worth *ten thousandths*, and consequently the *seven hundredths* amount to 70 thousandths ; since, likewise, a *tenth* is worth *ten hundredths* or *one hundred thousandths*, the *four tenths* will make *four hundred thousandths* : We can read this number in a more abridged manner, and say, *fifty three units*, *four hundred and seventy six thousandths*. Such is the usual way of reading a number composed of principal units and decimal parts. We might also read it thus ; *fifty three thousand*, *four hundred and seventy six thousandths* ; which will be easily understood, since every principal unit is worth *ten tenths*, *a hundred hundredths*, *a thousand thousandths*, and consequently every *ten* of units is worth *ten thousands of thousandths*. What we have here explained, is founded upon the general principle of Numeration, which is absolutely the same for decimal parts and for principal units.

If we had to read a number, containing no principal units, such as 0,4657, we should say, four thousand six hundred and fifty seven *ten thousandths*, because the units, expressed by the 7, are *ten thousandth parts*.

The 0 or cypher, amidst decimal parts, answers two purposes. 1st. It fills the place of the units wanting in the order where it is placed; then it serves to give a value ten times less to the figures at its right. In fact, suppose the number 45,308; the cypher indicates that there are no *hundredths*, and it gives a value ten times smaller to the 8; for if, omitting the 0, we wrote 45,38, the 8, which expressed *thousandths*, would now be *hundredths*.

It is plain, that the value of a number, containing decimal parts, would not be altered by the addition, at its right, of ever so many cyphers; and, consequently, if, at the right of decimal figures, there be cyphers, they may be safely suppressed.

REMARK. Since the *comma* serves to discern the decimal parts from the principal units, it is obvious that by moving this comma one fig. lower to the right, or higher to the left, we make this number ten times greater or smaller. Let us take, for instance, the number 534,728; if we move the comma one fig. to the right, and write 5347,28, the 7, which was of *tenths*, will become *units*; the 4, which expressed *units*, now expresses *tens*; the 3, which was of *tens*, is *hundreds*; the 5, which was *hundreds*, becomes *thousands*: and, on another hand, the 2, which expressed *hundredths*, is now of *tenths*; the 8, from *thousandths*, becomes *hundredths*. The moving of the comma, has then rendered every part of the number, and consequently the whole number, ten times greater than it was.—On the contrary, by placing the comma one fig. higher to the left, and writing 53,4728, we make the whole number ten times less, since every one of its parts, decreases in a tenfold proportion.

By a similar reasoning, it is easy to see, that, by moving the comma two, three, four, &c. fig. lower

to the right, we should make the number, 100, 1000, 10000, &c. times greater, and inversely if we moved it to the left.

It is to be wished that the pound, fathom, &c. had been divided into decimal parts ; arithmetical calculations would thereby have been greatly simplified.

A number is called *abstract*, which designates no kind of things, such as twice, three times, four times, &c. we call *concrete* a number applied to things, such as 25 fathoms, 50 pounds. This last kind of number is also called *incomplex*, when, as in the former example, it contains but units of the same denomination, and *complex*, when it contains units of various denominations, but reducible to the same, such as 15 fathoms 4 feet 6 inches—54£. 12s. 6d.

All the operations performed upon numbers, are reduced to four, viz. *adding*, *subtracting*, *multiplying*, and *dividing*. As the process of these operations is not the same for complex and incomplex numbers, we shall treat of them separately.

ANNOTATION.

For brevity's sake, it has been agreed to make use of certain signs ; viz.

$+$ which signifies *more*: $3+2$, signifies 3 *more* 2.

$-$ - - - - *less*: $9-4-1$, means 9 *less* 4 *less* 1.

$\times$ signifies *multiplied by*: 3×4, or 3 *multiplied by* 4.

$=$ signifies *equal to*: $4+3=5+2$, or 4 *more* 3, *is equal to* 5 *more* 2.

ADDITION

OF INCOMPLEX NUMBERS.

ADDITION is an operation directed to find out one number equal to several other numbers taken together, and the number, resulting from this operation, is called *the sum.*

The process consists, 1st. in placing the numbers one under another, in such a manner that the units be exactly under the units, the tens under the tens, hundreds under hundreds, and so on; and thus all the figures of the same kind of units form a vertical column. 2dly. Adding the figures of each column, beginning by that of simple units; and, if in the sum of any of the columns you find tens, you only write under that column the units belonging to it, and keep the tens to add them to the following column. An example will illustrate this theory.

3794206	*Numbers to be added together.*
48073	
259070	
643	
4101992	*Sum.*

Adding up the figure of the first column, I find 12 for the sum of the units in the four numbers; then I have written 2, and kept the *one ten* to join it to the column of *tens*; the sum of which having been found to be 19, I write 9 under it, and keep one *ten of tens* or a *hundred,* to add it up to the column of *hundreds*; proceeding on in the same manner, I have found 4101992 for the sum of the four numbers, which must evidently be right, since it is composed of the sums of all the units, tens, hundreds, &c. of those four numbers.

The proof of Addition is done by another addition executed in a contrary direction to the former; that

is to say, if you have proceeded *downwards* in the first operation, you must go *upwards* in the *proof*.

All that we have said, applies, without any exception, to numbers containing decimals.

Here you see that the disposition of numbers is always such, that the units of the same kind be placed the ones beneath the others.

	563,....	*numbers to be cast up.*
	8621,05..	
	4,8934	
	0,236.	
Sum	9189,1794	

Instead of the *dots* which mark the vacant places at the right of the numbers, we might have put *cyphers* without changing the value, the *comma* shewing that these are *decimals*.

OF SUBTRACTION.

SUBTRACTION is an operation, by which we find out the excess of one number above another; this excess is called *the difference*.

To proceed to the operation, you write first the greatest number, then place under it the smaller, so that the units correspond to the units, the tens to the tens, &c. Now, nothing would be easier than to find the excess of the larger over the inferior number, if all the fig. of the former were greater than the corresponding ones in the latter, but commonly it being not so, we shall see, by an example, how to perform every kind of subtraction.

From - - - -	56000940378
Subtract - - -	7092734287
Difference - -	48908206091

Having placed the two numbers the one under the other, I have subtracted the units from the units, saying; from 8 take 7, remains 1, which I have written underneath: now coming to the tens, and trying to take 8 from 7, I see it cannot be; then I have had recourse to the following fig, at the left, from which I have *mentally* borrowed one unit, which is a hundred, and consequently worth ten tens; I have joined these ten tens to the 7 of the 2d. figure, and from these 17 taking 8, I have 9 left, which I set down. (Remark that in order not to forget that I have borrowed a unit from the next figure, I have put a dot over this figure.) The 3, by this borrowing, being now worth only 2, I take 2 from 2 and nothing remains; I write 0. Proceeding on, I have said from 0 taking 4 cannot be done; I have taken a unit from the next fig. and then from 10 taking 4, I have 6 left; then 3 from 3, 0 remains; and 7 from 9, remains 2. Here a new difficulty has occurred, because having 2 to subtract from 0, which is impossible, I cannot borrow from the next fig. which is also 0. I have been obliged to reascend up to the first fig. after the cyphers; I have taken a unit from the 6, which being supposed carried on the next cypher, is worth ten; but as I wanted a unit of this ten for the next cypher, I have writen a small 9 over the 0 next to the 6; and proceeding along, from a similar reasoning, I have also written a small 9 over the second 0; then arriving at the last, I have said from 10 take 2, remains 8; then 9 from 9 and 0 remains; then from 9 take 0, remains 9; from 5 take 7, it cannot; borrowing a unit from the next fig. from 15—7, it leaves 8; at last from 4 nothing, remains 4. It is very evident, that 48908206091, is the difference between the two given numbers, since it is formed of the difference between each of their corresponding parts.

Subtraction is proved by Addition, viz. by adding the smaller number to the difference; for, if the ope-

ration be right, this sum must necessarily be equal to the greater number.

Example.	37540063
	27634054
Difference,	9906009
Proof,	37540063

The same process is perfectly applicable to numbers containing decimal parts.

First Example.

	754, 3845
	0, 0932
Difference,	754, 2913
Proof,	754, 3845

Second Example.

	843, 0000
	52, 7624
Difference,	790, 2376
Proof,	843, 0000

In the 2d. example, the decimals, altogether wanting in the larger number, have been supplied by cyphers, and the operation has been performed in the usual way.

OF MULTIPLICATION.

MULTIPLICATION is an operation by which we add a number to itself, as many times as another number contains units.

The first of these numbers is called *Multiplicand*, the second *Multiplier*, and the number, resulting from the operation, is called *Product*. The multiplicand and multiplier are also called the *factors* of the product.

It follows from the definition of Multiplication, that this operation is nothing but an addition ; for,

if we have two numbers to multiply one by the other, such as 24 and 6, 24 being the multiplicand, and 6 the multiplier, the question is then, to add 24 6 times to itself, which can be executed, as it is here, by the simple rules of Addition; and in reality 144 is certainly the product asked for, since it contains 24 6 times; but it is easy to see how tedious this process would be, if the multiplier were a considerable number.

$$\left\{\begin{array}{r} 24 \\ 24 \\ 24 \\ 24 \\ 24 \\ 24 \\ \hline 144 \end{array}\right.$$

Multiplication abridges this addition. The whole art consists in the use of Pythagoras's table, which must be got by heart; this table presents the products of all simple numbers, the ones by the others. A single example will show how simple, easy, and general, the rules of multiplication become, by the means of this table.

I place the multiplicand and multiplier one under the other; units under units; tens under tens, &c. and, having drawn a horizontal line under the multiplier, I successively multiply all the figures of the multiplicand by each figure of the multiplier, which gives so many partial products, which I write, the ones under the others, with the particular and essential attention, of *placing the first figure of each product, under the figure of the multiplier from which it proceeds*; then I cast up all the partial products, the addition of which, gives the total product asked for. Such is the process to be followed in all cases, of which I subjoin the explication.

EXAMPLE.

50794	*multiplicand.*
6033	*multiplier.*
152382	1st *partial prod.*
406352.	2nd do.
00000..	3d do.
304764...	4th do.
308979902	*total product.*

To get the first partial product, I say 3 times 4 are 12, I write 2, and keep, as in addition, the ten, to

join it to the second product, which I obtain by saying; 3 times 9 are 27, and 1 (brought from the first column) are 28 : I set down 8 and carry 2; then 3 times 7 are 21, and the 2 brought forward are 23; I write 3 and carry 2; 3 times 0 is 0, and 2 carried, are 2, which I set down; finally, 3 times 5 are 15, which I write down. This being done, it is evident that this first partial product contains 3 times the multiplicand, since it is formed of each part of the multiplicand taken 3 times.

By a similar process, it is easy to show, that the second partial product, as it is written, contains the multiplicand 8 times; but we must observe, that 8 being a number of tens, indicates that we must add the multiplicand to itself, not merely 8 times, but 8 tens of times, which obliges us to render this product 10 times greater; now, this is easily done, by writing the units of this product in the row of tens, whereby the tens have become hundreds, the hundreds thousands, &c. which consequently has rendered the total number ten times greater.

What we have said respecting the product of the multiplicand by 8, being applicable to all the other partial products, shows why we must write *the first figure of each partial product under the figure of the multiplier from which it proceeds.*

Now, casting up all the partial products, their sum must evidently be the total product asked for.

The 1st. partial product contains the multiplicand - - - - - - - - - -	3	times
the 2d. contains it - - - - - -	80	
the 3d. - - - - - - - - -	00	
the 4th. - - - - - - - - -	6000	
The total sum of these products will then contain it - - - - .	6083	times.

Then the number 308979902 is the product sought for.

REMARK. If both the multiplicand and multiplier, or either of them, were terminated by cyphers, the multiplication might be abridged by omitting the cyphers in both numbers; but then you must write, at the end of the product, as many cyphers as there are in both factors.

EXAMPLE.—Be it proposed to multiply 6370 by 5400; I only write - - - - -

637	*multiplicand.*
54	*multiplier.*
2548	
3185	
34398	*product.*

The multiplication being done, I add three cyphers to the product, which gives me 34398000 for the total product of the two given numbers. In fact, by cutting off the two cyphers of the multiplier, I had rendered this number one hundred times smaller; then the product must contain the multiplicand one hundred times less; but, this multiplicand, having itself become ten times smaller by the suppression of the cypher, it follows, that the product contains 100 times less a multiplicand 10 times too small, then this product must be 100 times 10 times, or one thousand times too small; I must then make it one thousand times greater, which is done by the addition of three cyphers.

THE MULTIPLICATION OF DECIMALS.

The Multiplication of numbers containing Decimals, is done the same as if there were none; but afterwards, you separate, at the right of the product, with a comma, as many figures as there were decimals in the two factors.

Example. Be it proposed to multiply 57,63 by 2,438, write the two numbers as if they had no decimals. The product being ascertained, separate 5 figures with the comma; the reason of this proceeding, is found by a reasoning similar to that of the preceding remark.	5763 2438 ——— 46104 17289 23052 11526 ——— 140,50194

PROOF OF MULTIPLICATION.

The proof of Multiplication is commonly done by Division; but, for incomplex numbers, it is often more expeditious, and equally sure, to do it by Multiplication itself; viz. by making the multiplicand multiplier, and *vice versa.*

Example.	Proof.
325	46
46	325
1950	230
1300	92
	138
14950	14950

Note. The proof by multiplication, is more expeditious, when the multiplicand and multiplier have a number of figures nearly alike.

We do not speak of the proof by 9, because it is in some cases deficient and faulty.

DIVISION.

DIVISION is an operation, by which is found how many times a number is contained in another.

The greater of the two numbers is called *Dividend*; the smaller, *Divisor*, and the result of the operation, *Quotient*. Both dividend and divisor are also denominated the two members or terms of a division.

When a division is to be only indicated, it is written thus, - - -

$$\frac{24 \textit{ dividend.}}{6 \textit{ divisor.}}$$

but when it is to be executed, the two members must be disposed in this manner, and the quotient placed under the divisor.

$$\textit{Dividend } 24 \left| \begin{array}{l} 6 \textit{ divis.} \\ \hline 4 \textit{ quot.} \end{array} \right.$$

It follows, from the definition of Division, that this operation can be done by *Subtraction*. If we had, for instance, 24 to divide by 6, the question would then be, to find how many times 6 is contained in 24, which can be known by subtracting 6 from 24 as many times as can be, and the number of subtractions will be the quotient sought for, which will be 4, as it is seen by the number of subtractions made here:

But such a process would be too tedious if the divisor was contained a great number of times in the dividend. The art of Division is to shorten this operation, which abbreviation consists particularly in the use of Pythagoras's Table, which shews how many times a divisor of one figure is contained in a dividend, composed, at most, of two.

$$\left.\begin{array}{r} 24 \\ 6 \\ \hline \end{array}\right\} \text{1st } \textit{subtraction.}$$
$$\left.\begin{array}{r} 18 \\ 6 \\ \hline \end{array}\right\} \text{2nd } \textit{subtraction.}$$
$$\left.\begin{array}{r} 12 \\ 6 \\ \hline \end{array}\right\} \text{3d } \textit{subtraction.}$$
$$\left.\begin{array}{r} 6 \\ 6 \\ \hline \end{array}\right\} \text{4th } \textit{subtraction.}$$
$$0$$

This being understood, we shall distinguish only two cases in Division, the one when the divisor contains but one figure, whilst the dividend is composed of ever so many; the other, when both members consist of several figures.

The method to be followed in both cases, is, to consider the given dividend as composed of several partial dividends, in which the divisor be contained the fewest times possible, in order to have always but one figure to write in the quotient. The operation is commenced by the left of the given dividend, in order that the excess of the first partial dividend, over the product of the divisor by the quotient, may be added up to the second partial dividend, and so on; which could not be done, without much trouble, if the operation began by the right of the dividend. This will be illustrated by the following example, and by the explications we shall give of our proceedings.

DIVISION.—First Case.

```
9471 | 7
.... |-----
7    | 1353
--
24
.21
---
.37
 35
 ---
 .21
  21
  ---
   0
```

I remark, first, that the first figure, at the left of the dividend, containing the divisor, must be the first partial dividend; then I say, in 9 how many times 7? 1; I write 1 in the quotient; and, to find the excess of this first partial dividend, over the product of the divisor by the quotient, I say, once 7 is 7, which I write under the 9, and taking 7 from 9 I have 2 left; I take down the second figure of the dividend, which I write by the remainder 2, and then the second partial dividend is 24; I then

say, in 24 how many times 7? 3 times; I write 3 in the quotient, and then, to find the excess of the dividend, I say, 3 times 7 is 21, which I set down under 24, then subtracting, I have 3 for the difference; by this 3 I take down the third figure of the dividend, and I have, for my third partial dividend, 37; operating as above, I find for quotient 5, with the remainder 2; finally, I take down 1, and 21 is my last partial dividend, which gives 3 for quotient, and nothing remains; therefore I say, that 1353 is the quotient asked for, or that it represents the number of times the divisor is contained in the dividend, since it is the assemblage of all the quotients of the partial dividends; and the better to be convinced of it, you need but observe, that the first partial dividend was not really 9, but 9000; therefore, when we have said, in 9 how many times 7? it is plain that we have made use of a dividend 1000 times too small, which of course must have given a quotient 1000 times too small; for a dividend 1000 times too small, must necessarily contain the divisor 1000 times too little; then the quotient must be made 1000 times bigger; and, instead of 1, we must write - - 1000; you see, likewise, that the second partial dividend, being a number of hundreds, the quotient 3 should also express hundreds, and ought to have been written - - - - - - 300; the third partial dividend, being a number of tens, its quotient 5 amounts to - - - - 50.
In fine, the fourth partial dividend, being a number of units, has for quotient - - - 3.
Casting up these partial quotients, we find the total quotient to be - - - - - - 1353.

EXPLICATION.

Division.—Second Case.

```
753 209 | 368
 ....   |-----    281
736     | 2046    ---
----              368.
 1720
 1472
 ----
  2489
  2208
  ----
   281
```

Here you see, that the divisor, being a number of hundreds, the first partial dividend must be composed of at least 3 figures; taking then the 3 first figures on the left of the dividend, we have said, in 753 how many times 368? it is twice: I have written 2 in the quotient; and, multiplying the divisor by this quotient, in order to subtract it from the dividend, we had 736, which, subtracted from the partial dividend 753, gives 17 for remainder, which I write, and close by it the 2 which I take down, to form the second partial dividend: then I say, in 172 how many times 368? no times; I write 0 in the quotient; then proceeding, I take down a new figure, which gives 1720 for the third partial dividend, and I say, in 1720 how many times 368? or, as it is not easy to find the ratio between two numbers when they are of some extent, I content myself, by way of a trial, with comparing the units of the highest kind in the divisor, with the same in the dividend; here, therefore, comparing the hundreds in both members, I say, in 17 how many times 3? It gives 5 times: but, before I write down 3 in the quotient, I must examine whether what remains of the dividend will also contain 5 times the remainder of the divisor. In general it is enough to try the 2d. figure of the divisor; I say then, 5 times 6 are 30, (i. e. 30 tens) carrying mentally the 0 under the figure 2 of the dividend (which is also a number of tens)

I have retained 3; then, saying 5 times 3 are 15, and the 3 retained are 18, I see that 5 was too great, since 18 cannot be taken from 17, I have tried 4, which I have found right; I have then written it to the quotient; multiplying and subtracting, I have 248 for remainder; taking down the 9, I have had 2489 for my last partial dividend, which has given 6 for quotient, with the remainder 281, which I set down by the quotient, under the form of a division not executed, and which in reality can only be indicated, since a farther division is impossible.

N. B. Care must be taken to put a dot under each of the figures of the dividend, as you are taking them down to form the partial dividends, in order to avoid the oversight of taking down the same twice; these dots will besides indicate the number of figures which must be had in the quotient.

REMARK 1st. Division can always be shortened, when the *divisor* is terminated by a number of cyphers; with regard to the *dividend*, either it has a number of cyphers, equal or superior to that in the divisor, or it has none at all, or it has a smaller number.

In the 1st. case, you must strike off, in both members, an equal number of cyphers; which, as it is evident, will give the same quotient; for, by cutting off one, two, three, &c. cyphers, at the right of the divisor, you render this number 10, 100, 1000 &c. times smaller, and therefore capable of being contained that number of times more in the *dividend*: the quotient is then restored, if, at the same time you make the dividend proportionably smaller.

Example.—First Case.

$$548000 \;\big|\; 700$$

write

$$5480 \;\big|\; 7$$

The way to shorten division in the 2d. case, is to separate as many figures on the left of the dividend, as there are cyphers on the right of the divisor, which are also to be separated by a line; then make your division with the two members thus reduced; but the operation being done, you must take down, to the right of the remainder, the figures which had been severed at the right of the dividend.

Example. Second Case.

674	345	24	(000
..		28	2345
48			
194			24000
192			
2345			

In the present example we have followed the process indicated above, of which this is the explication; it is evident from what has been said on the first case, that taking 674 for dividend, and 24 as divisor, we have acted the same as if the total dividend had been 674000, and the divisor 24000; but as the number 345 is a portion of the given dividend, this portion must necessarily be added to the remainder 2, which, being a number of thousands by the rank it holds, the total remainder will be 2345; the exact quotient will then be, 28 $\frac{2345}{24000}$; for, since, with respect to the remainder of the division, we reinstate the dividend in its integrity, we must do the same with the divisor.

REMARK 2nd. The quotient of a division expressing the number of times the divisor is contained in the dividend, it is plain that the product of the divisor by the quotient, must give the dividend; thus, if I have 24 to divide by 4, the question is to find a number, which, multiplied by 4, be equal to 24; this number must then be 4 times smaller than 24, or the fourth of 24; whence it follows that to divide a

number by 2, 3, 4,.....10, &c. is the same as to take the half, the third, the fourth.....the tenth part of that number.

DIVISION OF DECIMALS.

The division of numbers which contain decimal parts, can always be executed without difficulty by using a general method, which consists in having the same number of decimals in both members, which will be obtained by adding cyphers at the right of the member which contains fewer decimals; then omitting the comma, make the division, which will give the quotient asked for : for, it is evident, that rendering, by the suppression of the comma, both dividend and divisor the same number of times greater, we change nothing in the quotient.

EXAMPLE. Be it proposed to divide 34,7 by 3,506; we must write - - 34700 | 3506

It sometimes happens that the dividend is not exactly divisible by the divisor, and then the true quotient is to be obtained only within a unit; yet, in many cases, it is necessary to obtain a more exact quotient; the decimals furnish us with an easy mean of obtaining it within as approximate a degree as can be wished, viz. within a tenth, a hundredth, a thousandth, &c. we need for that but add as many cyphers at the right of the dividend as we wish to have decimals in the quotient.

EXAMPLE. Be it asked for the quotient of 459 by 23, within a hundredth part of unit.

Since we must have two decimals in the quotient, we must then make the dividend a hundred times

too great, by writing two cyphers to the right of that number, and the division to be executed will be - - - - - - -

45900	23
23	19,95
229	
207	
220	
207	
130	
115	
15	

That total quotient 1995 which has been obtained, is a hundred times too great, since we had made the dividend a hundred times too great; the quotient must then be rendered a hundred times smaller, which, as has already been said in Numeration, is effected by separating with a comma two figures on the right of that number, and it gives for the true quotient, nineteen units and ninety-five hundredths; here it is evident that the quotient is right within one hundredth, for, if instead of 5 hundredths, we had written 6, this figure would be too high.

Proof of Division and Multiplication.

When a division is done right, the product of the divisor by the quotient, added to the remainder, if there be any, must be equal to the dividend. Thus, the proof of division is done by multiplication, and consequently multiplication is also proved by division; for, dividing the product of a multiplication by one of its factors, the question must be the other factor.

OF FRACTIONS.

A fraction is a quantity less than a unit; if then we consider the unit divided into two, three, four,

&c. equal parts, the half, the two thirds, the three fourths, of the principal unit will be fractions.

Two numbers are necessary to express a fraction, they are written one under the other, and separated by a line, just as a division when it is only indicated; thus $\frac{3}{4}$ represent a fraction, which we read, three fourths; it means the three fourths of a unit. The superior number is called *numerator*, and the inferior *denominator*. This one shews into how many equal parts the principal unit is supposed to be divided, and the numerator expresses how many of these parts are taken.

This is the most direct manner of considering a fraction; but there is another not less important, as it easily accounts for several operations to be performed on fractions: it is to consider the two terms of a fraction as the two members of a division, whereof the numerator is the dividend, and the denominator the divisor. But we must be fully convinced that this new manner of considering a fraction, leads to the same result as the former: let us take the fraction $\frac{3}{4}$ for example; we must prove, that, dividing the unit into four equal parts, and taking three of them, we shall have the same result, as if taking three of the same units, we divided them by four. This proof can be rendered sensible by an application.

Let us suppose that the principal unit be the pound, which is worth 20 shillings: from the first manner of considering a fraction, we must divide the pound into four equal parts, each of which will be 5s. and then taking three of these parts, we shall have 15s. Now, if 3, represent a dividend, it must be considered as 3 units, or 3£. or 60s. but 60s. divided by 4, give 15s. we have then the same quotient as above; we may, therefore, indifferently consider fractions in either of these two manners.

Fractions are subject to the same operation as whole numbers; they may be added, subtracted, multiplied, or divided.

ADDITION OF FRACTIONS.

Suppose several fractions such as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, which we must add together; this operation is indicated as follows: $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$.

We shall give out the process first, and then the explanation. Begin with bringing the several fractions to the same denominator, which is done by multiplying both terms of each fraction by the product of the denominators of all the others. By this operation, the above fractions become $\frac{24}{48}+\frac{36}{48}+\frac{40}{48}$. They all have the same denominator, which could not fail, since each denominator is the product of all the denominators of the above fractions. Now, to add them up, take the sum of all the numerators, for it is evident that $\frac{24}{48}+\frac{36}{48}=\frac{60}{48}$, and that $\frac{60}{48}+\frac{40}{48}=\frac{100}{48}$, the last fraction being the sum of the three proposed fractions.

Before I proceed, I observe, that $\frac{100}{48}$ is not properly a fraction; for, from the definition, a fraction is a quantity less than a unit; but we call this a fractionary number, and, if the division be effected, we shall have for result two units, more the fraction $\frac{4}{48}$.

Now, let us analyse the foregoing operation: I said, first, that we must bring all the fractions to the same denominator. Why so? because we cannot add together things of various kinds; and it is the denominator which designates the kind of the fractionary unit. No demonstration is necessary to show that it is impossible to add $\frac{3}{4}$ with $\frac{5}{7}$, whereas there is no difficulty in adding up $\frac{2}{7}$ with $\frac{4}{7}$, which evidently gives $\frac{6}{7}$: but in order to prove in a sensible manner what I have advanced, viz.: that it is the denomina-

tor of a fraction which designates the kind of the fractionary unit, let us take an example, and suppose that the fathom be the principal unit, and that we have to add the two fractions $\frac{5}{6}$ and $\frac{7}{72}$; the fathom being equal to six foot, it is plain that the $\frac{5}{6}$ express five foot; likewise the fathom being equal to 72 inches, the $\frac{7}{72}$ of a fathom will be worth 7 inches; consequently in the question, as above stated, we had feet to add with inches, which is impossible, and shows very sensibly that it is the denominator which marks the kind of the fractionary unit. With respect to the operation for bringing the fractions to the same denominator, it is evident, that by multiplying both numerator and denominator by the same number, we do not alter the value of the fraction, because the numerator and denominator represent the two terms of a division, and we know that the quotient is not changed, if after making the dividend a certain number of times greater, we also make the divisor greater in the same proportion.

From what has been said it follows, that in practice, the addition of fractions consists in multiplying the numerator of each fraction by the product of the denominators of all the others; then adding all these numerators, and dividing that sum by the product of all the denominators.

If we had a whole number to add with one or several fractions, this particular case would be brought under the general rule, by supposing that the whole number has 1 for denominator.

Example. $\frac{5}{1}+\frac{2}{3}+\frac{1}{4}$, reduced to the same denominator, give me $\frac{60\times 8+3}{12}=\frac{71}{12}$, a fractionary number resulting from the addition of the whole number 5, with the two fractions $\frac{2}{3}+\frac{1}{4}$.

Hence, to add a whole number with one fraction only, you must multiply the whole number by the denominator of the fraction, to this product add the numerator of the same fraction, and divide the whole of it by its denominator.

N. B. That the general rule just given to bring fractions to the same denominator, would induce into lengthy calculations, which may often be avoided by an abridged method which comprises three different cases.

First Case. When the greatest denominator among all the fractions, is found to be a multiple of each of the other denominators.

EXAMPLE. $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{2}{3}+\frac{1}{12}$; 12, which is the greatest denominator, may be taken for the common denominator; in fact, by multiplying the denominator of the first fraction by 6, that of the second by 3, of the third by 2, &c. 12 would be the common denominator; but then in order not to change the value of the fractions, you must multiply their numerator by the same number by which you have multiplied their denominator; thus the fractions will become $\frac{6+9+10+8+1}{12}=\frac{34}{12}$, result very simple; which by the general method would have been found $\frac{4896}{1728}$.

Second Case. When the greatest denominator being taken 2, 3, 4, &c. times, gives a number multiple of the denominator of each fraction.

60

EXAMPLE. $\frac{1}{5}+\frac{3}{20}+\frac{5}{6}+\frac{7}{15}$; I see, that taking three times the greatest denominator 20, I have 60 which I can take for common denominator, since it is a multiple of all the denominators. Write this

number above the fractions, as you see in the example, in order to facilitate the operation to be performed on the numerators. The proposed fractions, having 60 for common denominator, are reduced to

$$\frac{12+9+50+28}{60}=\frac{99}{60};$$

by the general method the result would have been found $\frac{1584}{960}$*.

Third Case. When the denominators of fractions, though not falling under either of the two preceding cases, are nevertheless composed of some common factors.

Example. $\frac{3}{4}+\frac{5}{12}+\frac{2}{9}+\frac{4}{45}+\frac{7}{20}$; I write these fractions under the following form, resolving their denominators into factors as nearly alike as possible,

$$\overline{4\times3\times3\times5}$$

$$\frac{3}{4}+\frac{5}{4\times3}+\frac{2}{3\times3}+\frac{4}{3\times3\times5}+\frac{7}{4\times5}:$$

Now, in order to have an abridged expression of the denominator, I compose it as it is seen on the top of these fractions, taking first the denominator of the 1st fraction such as it is: then the 3 of the 2d. then one 3 of the 3d. then the 5 of the 4th, and none of the factors of the denominator of the 5th, because they had already been written.

It is evident, that the number expressed by $4+3+3+5$, which is equal to 180, may serve as a common denominator, since it is (as may be seen by its factors) a multiple of all the denominators. Multiplying, now, each of the numerators, by the same number by which its denominator should be multi-

plied to produce the common denominator 180, the fractions become $\frac{135+75+40+16+63}{180}=\frac{329}{180}$ which, by the general method, would have been $\frac{710640}{388800}$.

SUBTRACTION OF FRACTIONS.

This operation presents no difficulties. The fractions, as in addition, must be brought to the same denominator, and the subtraction be executed on the Numerators.

EXAMPLE. Suppose $\frac{2}{5}$ are to be subtracted from $\frac{3}{4}$, which is represented in this manner $\frac{3}{4}-\frac{2}{5}$; brought to the same denominator, these two fractions become $\frac{15-8}{20}$; then the subtraction being effected on the numerators, will give $\frac{7}{20}$.

Another Example, more complex: $\frac{1}{2}-\frac{2}{3}+\frac{5}{6}-\frac{1}{4}$, equal to $\frac{72-96+100-36}{144}=\frac{172-132}{144}=\frac{40}{144}$.

If we had a fraction to subtract from a whole number, we should multiply the whole number by the denominator of the fraction, subtract from it its numerator, and divide the result by its denominator. This process will be easily understood, by supposing the unit as a denominator to the whole number.

MULTIPLICATION OF FRACTIONS.

In order to multiply one fraction by another, multiply the two numerators together, and likewise the two denominators.

Example. Suppose the fraction $\frac{3}{4}$ is to be multiplied by $\frac{2}{5}$ which is thus written $\frac{3}{4}\times\frac{2}{5}$. I multiply 3 by 2, and 4 by 5; and the product is $\frac{6}{20}$. To prove it, let us first take the simplest case, and suppose we have not $\frac{3}{4}$, but simply 3 to multiply by $\frac{2}{5}$, or $3\times\frac{2}{5}$; without the least notion of the process to be followed, any one will be apt to multiply first 3 by 2; but I observe this product to be too great; for it was not by 2 I had to multiply, but by 2 divided by 5, or by the 5th part of two units; this product of 3 by 2, is then 5 times too great; to reduce it to its proper value I must then divide it by 5, and write $\frac{3\times2}{5}$.

We shall here observe, that to multiply a whole number by a fraction, we must multiply that number by the numerator of the fraction, and divide the product by the denominator of said fraction. Let us now return to the question proposed; I say now, that it was not 3 I had to multiply by a fraction, but 3 divided by 4, or the 4th part of 3; therefore, in the expression $\frac{3\times2}{5}$, the numerator is 4 times too great; I must then, in order to bring down this fraction to its true value, make its denominator 4 times greater, and then write $\frac{3\times2}{5\times4}$; we must then

multiply numerator by numerator, and denominator by denominator.

DIVISION OF FRACTIONS.

The process in this operation, is quite inverse of the former, and is accounted for by the same explications.

EXAMPLE. Be this fraction $\frac{3}{4}$ to be divided by $\frac{2}{5}$; write it $\frac{3}{4}$ d. $\frac{2}{5}$. Here you have to multiply the numerator of the dividend, by the denominator of the divisor, for the numerator of the new fraction, and reciprocally for the denominator of the said new fraction; the quotient will then be $\frac{15}{8}$. In order to prove it, let us take the simplest case, and suppose we have not $\frac{3}{4}$, but only 3 to divide by $\frac{2}{5}$, or 3 d. $\frac{2}{5}$; what naturally occurs, is, to divide 3 by 2, which gives $\frac{3}{2}$: but I observe this fraction is too small, for its denominator is too great, since it was not by 2 I had to divide, but by 2 divided by 5, or by the 5th part of 2. I must then, to re-establish the value of this fraction, make it 5 times bigger by multiplying its numerator by 5. I will then have $\frac{3+5}{2}$; then the division of a whole number by a fraction, consists in dividing that number by the numerator of the fraction, and multiplying this result by its denominator. But now again I say; It was not (by the state of the question) 3 I had to divide, but 3 already divided by 4, or the 4th part of 3; therefore, in the expression $\frac{3\times5}{2}$, the numerator is 4 times too great; I must then, to make it 4 times less, make the denominator 4 times larger, and write $\frac{3\times5}{4\times2}=\frac{15}{8}$ which was to be demonstrated.

FRACTIONS OF FRACTIONS.

Be it proposed to take the $\frac{2}{3}$ of $\frac{4}{5}$; this is what is called fraction of a fraction, as likewise to take the $\frac{2}{3}$ of the $\frac{3}{4}$ of $\frac{5}{6}$; or of a still greater number of fractions.

These questions, at first sight, seem somewhat puzzling, yet in itself, nothing is more simple than the process to be followed in their solution. Multiply all the numerators together, and likewise the denominators; the result of the first question will then be $\frac{8}{15}$, and of the second $\frac{30}{72}$. In order to prove it, let us stick to the 1st example. It is asked for the $\frac{2}{3}$ of $\frac{4}{5}$; that is; we must take the $\frac{2}{3}$ of a quantity expressed by $\frac{4}{5}$, or divide this last quantity into three equal parts, and take two of them. Now, to divide $\frac{4}{5}$ by 3, is to render this fraction 3 times smaller, and consequently multiply its denominator by 3, which gives $\frac{4}{5 \times 3}$; but we must take this fraction twice, the exact result will then be, $\frac{4 \times 2}{5 \times 3}$; therefore the two numerators must be multiplied by each other, and so also the denominators.

For the 2d case, or any greater number of fractions, the demonstration would be the same; for first, in order to find out the $\frac{2}{3}$ of $\frac{3}{4}$, we must multiply $\frac{3}{4}$ by $\frac{2}{3}$, which gives $\frac{6}{12}$ and reduces the question to the taking the $\frac{6}{12}$ of $\frac{5}{6}$, case similar to the former, the result of which is $\frac{30}{72}$, as we had advanced it.

REDUCTION OF FRACTIONS.

It frequently happens that both the numerator and denominator of a fraction are large numbers, as it is

the case in most divisions, where the quotient commonly contains a fraction; some of these fractions can be reduced to a very simple expression; there are two methods to accomplish it; the way of trying, and that of the greatest common divisor.

Here is the process to be followed in the first case, which will be easily understood by means of an example.

$$\frac{1260}{1764}=\underbrace{\frac{630}{882}}=\underbrace{\frac{315}{441}}=\underbrace{\frac{105}{147}}=\underbrace{\frac{35}{49}}=\underbrace{\frac{5}{7}}$$

Reductions, 1st. 2nd. 3d. 4th. 5th.

Whenever the numerator and denominator are both even numbers, they can both be divided by 2, or the half of them be taken, as we have done here, to effect the first reduction; when 2 can no longer be a common divisor, we have recourse to 3, and divide by 3 as often as possible; after 3 we try 5; It would be useless to try 4, because 2, which is an under multiple of 4, being no more contained in it, 4 cannot be; 5 being exhausted, it is not necessary to try 6, as it is a multiple of 3, we pass to 7, then 11, 13, &c. taking only the *primary numbers*, which can be divided but by itself or by 1. It would be tedious to carry your trials above 11, as the primary numbers above it, not being contained in the multiplication table, this reduction would require a complicated division.

NOTE. To facilitate this kind of reduction, it is important to know, that a number is exactly divisible by 3, whenever the sum of its figures, added up together as simple units, is an exact multiple of 3; and that a number can be divided by 5, only when it is terminated by 0 or by 5.

GREATEST COMMON DIVISOR.

The above method, though only a trying, is preferable to the general rule, whenever the two numbers are not very large; if they be, take the following easy method.

Divide the denominator of the fraction by its numerator; if the quotient leaves no remainder, the numerator itself will be the greatest common divisor. If there be a remainder, divide by this the numerator; if the quotient be exact, then this remainder will be the common divisor; should there be a new remainder, the former remainder must be divided by the second, and so on, 'till your division leaves no remainder; then the last divisor is the greatest common divisor, which will be used to divide by it both terms of the fraction, and the two quotients will be the two terms of the fraction reduced to its lowest expression.

EXAMPLE. Reduce the fraction $\frac{2431}{3315}$.

	1	2	1	3
3315	2431	884	663	221
2431	1768	663	663	
884	663	221	000	

I have divided the denominator by the numerator, then the numerator by the first remainder 884, then this by the 2d remainder 663, the 2d by the 3d 221, which is found to be the greatest common divisor, for the division leaves no more remainder. Now, dividing the numerator of the fraction by 221, we find 11 for exact quotient, and dividing the denominator by the same, we find 15, then $\frac{11}{15}$ is the value of the fraction reduced to its lowest terms.

NOTE. In the above operation, we have written

the quotients above the divisors, in order to facilitate the process.

Demonstration of the Preceding Rule.

We have to prove two things; first, that the number 221, thus found, is necessarily the common divisor of both terms of the fraction, or that it must divide both exactly. Secondly; that it is their greatest common divisor.

I say first, that 221 is a common divisor, for we see that it divides 663 without remainder; and as it does also divide itself exactly, it must likewise divide 884, which, in the principles of division, is equal to 663×221; but 2431, being, according to the same principles, equal to $884 \times 2 + 663$, it is plain, that as 221 divides exactly both 884 and 663, it will also divide 2431, which is a compound of both. A similar reasoning will shew, that 221, will divide, without remainder, 3315, which is equal to 2431×884, which is divided exactly by 221.

I say now, that the two terms of the fraction, cannot have a greater common divisor than 221; for, it is evident, in the first place, that the greatest common divisor of the two terms of a fraction, cannot be greater than the numerator of that fraction, which is the smaller of those two terms. Now, the numerator, in this case, not being an exact divisor of the denominator, but having a remainder, I say, that the common divisor cannot be greater than this remainder; for we can write the fraction under this form: $\dfrac{2431}{2431 \times 884}$; now it is obvious, that the greatest common divisor of these two terms, cannot be greater than 884, for it must be such as to divide both 2431 and 884; and 884 cannot be divided by a number

greater than itself. Having then tried this number, it has been found to divide 2431 with a remainder 663, and 2 for a quotient; then, in the place of the two terms of the fraction, we may write:

$$\frac{884\times2+663}{884\times2+663+884}$$; then we see, by a discourse similar to the former, that 663 is the greatest common divisor that can be tried. It is therefore easy to convince one's self that the greatest common divisor that can be used, is the divisor by which a quotient is obtained without remainder; consequently 221, being in that case, is the greatest common divisor of the proposed fraction.

COMPLEX NUMBERS.

A number is called complex, which contains units of different kinds, but all reducible to the same kind. Thus 12£. 6*s*. 8*d*. is a complex number, because it can be all reduced to pence, its lowest denomination. 15 fathoms 4 feet 7 inches, is likewise a complex number, reducible to inches.

ADDITION OF COMPLEX NUMBERS.

The addition of complex numbers is effected on the same principles as that of incomplex ones, by placing the units of the same kind, the ones beneath the others, so as to form vertical columns, and the addition is begun by the lowest units. An example will make this obvious.

The sum of pence has been found to be 26; we have set down 2, and carried 2 to the column of shillings, (24 pence making two shillings.) Passing to the 2d column, we have said; 2, brought forward, and 8, are 10, and 2 are 12; set down 2, and carry 1, which, joined to the tens of shillings, give 3 tens; and, as 2 tens of shillings make a pound, we have set down 1, worth one ten, to the column of shillings, and carried 1, worth one pound, to that of pounds, and then continued the operation as for incomplex numbers.

£	s.	d.
345	8	9
442	12	11
2453	10	6
£.3341	12	2

Subtraction of Incomplex Numbers.

This operation needs only be exposed to be understood.

Not being able to subtract 5 feet from 2 feet, we have had recourse to the superior units, and borrowed from the figure 9 one unit, worth 6 feet, which, joined to 2, gives 8; and then, from 8 take 5, and 3 remains; the rest as in common subtraction.

Subtraction.

	Fath.	*ft.*	*Inc.*
	329	2	9
	235	5	7
Differ.	93	3	2

Multiplication of Complex Numbers.

This is one of the most complicated operations in arithmetic. It comprises many cases which can be resolved by various abridged methods, very convenient in practice: but, as they all fall under the general method, we shall content ourselves with explaining it here, and making the application of it to the most difficult case.

Be it proposed to multiply 325£. 17*s.* 11*d.* by 659 fathoms 5 feet 10 inches.

MULTIPLICATION.

	£.	s.	d.	
	325 ..	17 ..	11	
	659 *fath.*	5 *feet.*	10 *inch.*	
	2925.			
	1625			
	1950			
2*s.* - - -	65	18*s.*	0*d.*	
14 - - - - -	461	6	0	
1 - - - -	32	19	9	
6*d.* -	16	9	6	
3 - -	8	4	9	72.
2 - -	5	9	10	
3 *feet,* - -	162	18	$11\frac{1}{2}$	36
2 - - -	108	12	$7\frac{2}{3}$	48
6 *inch.*	27	3	$1\frac{11}{12}$	66
3 -	13	11	$6\frac{23}{24}$	69
1 -	4	10	$6\frac{23}{72}$	23
	215082£.	3*s.*	$11\frac{13}{36}$*d.*	$\frac{242}{72}$

Numerators of the fractions brought to the same denominator, 72.

It may be asked how it happens, that pounds are to be multiplied by fathoms, this proposition implying a seeming absurdity. It does indeed, but we must observe;

First; that in every multiplication, the multiplier is always to be considered as an abstract number, since it only serves to indicate the number of times the multiplicand is to be added up to itself; and, if it is used to designate particular units, it is only to know the ratio of the fractionary parts of that number with the principal unit; *here the fractionary parts are 5 feet 10 inches.*

Secondly. That the questions, which give occasion for complex multiplications, are almost all of the same nature, and can all be stated as in the annexed example, by saying; if a fathom, of a certain kind of work, has cost 325£. 17s. 11d. how much will cost 659 fathoms 5 feet 10 inches? To answer this question, it is evident we must repeat the 1st number as many times as the 2nd contains fathoms and parts of fathoms, that is, multiply the first by the second. Let us proceed, now, to the explication of the operation.

I have multiplied first 325£. by 659; then to multiply the 17s. by the same number, I have considered that 17s. was composed of 2, 14, and 1s. which I have written on the left of the partial products in one vertical column; the reason of this subdivision will soon be understood. It must always be made in this manner, that is to say; we must always begin by 2s. because this produce is obtained with great facility, nothing more being required for it, than doubling the figure of the units of the multiplier, writing them to the column of shillings, and carrying the other figures of the multiplier to the column of pounds.

Here, then, we have said; twice 9 are 18, which we have set down for shillings, and the 65 have been written in the column of pounds. This is the reason of that operation.

If I had 1 pound to multiply by 659, the product would be 659£; but as I have only two shillings to multiply by 659, and as two shillings are the 10th part of 1 pound, I must then take the 10th part of 659, considered as a number of pounds; It is what we have done; for 659£. being the same as 650£.+9£. It is evident that writing only 65, we have taken the 10th of 650; and besides 2s. being the 10th of 1£. the 10th of 9£. will be 9 times 2s. or 18s. This, well understood, will greatly facilitate the rest.

The product for 14s. is obtained by taking 7 times the product for 2s., since 14 contains 7 times 2, and we have said; 7 times 8 are 56, we have set down 6 and carried 5, then 7 times 1 is 7, and 5 carried, are 12, which are tens of shillings, and consequently £6. which have been carried; then continuing to multiply by 7, we have said; 7 times 5 are 35, and 6 carried are 41; we have set down 1 and carried 4; then 7 times 6 are 42, and 4 carried are 46, which have been written down. 1s. being the half of 2s. we have taken for the product by 1s. the half of the product by 2s. saying; the half of 6 is 3, the half of 5 is 2 for 4, and remain 1, which is £1. worth 20s. which, being joined to the 18s. have made 38, the half of which is 19.

Now for the multiplication of Pence, a general rule is, to decompose the number of pence in the multiplicand, into *aliquot parts* of 12, such as 2, 3, 4, 6.—We call *aliquot parts* of a number, another number which is contained in it an exact number of times.

We have decomposed 11d. into 3 parts; and since 6d. are the half of 1s. we have taken for product by 6d. the half of the product by 1s.—then 3d. being the half of 6d. we have taken the half of the last product; in fine 2d. being the third of 6d. we have taken the third of the product by 6d.

So far we have multiplied all the parts of the multiplicand by the principal units of the multiplier; let us now see how we operate with the fractionary parts of the multiplier.

We have decomposed 5 feet into 3 and 2 feet, which are *aliquot* parts of 6 feet, or of a fathom; and we have said; since a fathom costs £325 17s. 11d. It is evident that 3 feet will only cost one half of it; therefore to have the product by 3 feet, we have taken the half of the multiplicand; for the product

by 2 feet we have taken the third of the product by 1 fathom, or the third of the multiplicand.

In fine, we have also decomposed 10 inches into *aliquot parts* of 12 inches, or 1 foot, and we have said; 6 inches being the half of a foot, or the 4th of 2 feet, we have to take for 6 inches, the 4th part of the product by 2 feet; but it is to be observed, that, to perform this operation, when we have come to the pence, we have said, the 4th part of 7 is 1, and 3 remains, of which we must take the 4th, as well as that of the fraction $\frac{2}{3}$; then we have joined the whole number 3 to the fraction $\frac{2}{3}$, which has given $\frac{11}{3}$, the 4th part of which is $\frac{11}{12}$, which have been set down.

For the 3 inches, we have taken the half of the product by 6 inches; and in fine, for 1 inch, the third of the last product. Casting up, now, all the partial products, it is evident that we must have the total sum demanded.

Remark. At the right of the fractions in the above operations, you see a small square of a very convenient form, to add up together any number of fractions disposed in a column. Having found, by the abridged method given above, a common denominator for all those fractions, which is here 72, we have written it at the top, and under it the new numerators, the sum of which has been found 242, which, divided by 72, gives for quotient $3+\frac{26}{72}$; we have carried 3 to the units of pence, and reduced the fraction to $\frac{13}{36}$.

We set down another example, to give occasion to new observations.

Note. We have taken 25 as multiplier, and 326 as multiplicand in the multiplication of the principal units, in order to have one product less to write.

Multiplication.

	£.	s.	d.		
	25 ..	8 ..	1		
	326 *fath.*	3 *feet.*	2 *d.*		
	1630				
	652				
2*s.*	32	12*s.*	0*d.*		
6	97	16	0		
1	~~16~~	~~6~~	~~0~~		
1*d.*	1	7	2		
3 *feet.*	12	14	0	6	
1	~~4~~	~~4~~	~~8~~	~~8~~	
2 *inc.*	0	14	1	4.	4.
	8295£.	3*s.*	4*d.*		

Remark 1st. That, after obtaining the product for 8s. we have taken the product for 1s. which is called false product, because it is no part of the total addition, and serves only to lead to the following product; for, without it, it would have been difficult to find the product for 1d. which has been easily obtained, by taking the 12th of this false product.

A similar false product has been likewise made for 1 foot, in order to facilitate that for 2 inches; care must be had to bar the figures of such false products, which, as we have said, are not to be cast up with the rest for the total sum.

Remark 2d. That, in the product for 3 feet, instead of expressing by the fraction $\frac{1}{2}$, the half of a penny, we have supposed the penny divided, like the shilling, into 12 equal parts, and even we see below, in the sketch of the operation, that each of these new parts has been divided into other 12, which subdivisions by 12 are continued as often as necessary, in order to avoid fractions. This method is useful in

practice, tho' it must be agreed that there are certain numbers which would lead to infinite.

REMARK 3d. That, in the addition of the lower denominations, under the pence, no account has been taken of the units of the lowest kind, because their sum was under 6; and that, on the contrary, we have counted for 1 penny those of the next column, because their sum exceeded 6; this is received in practice in the settling of accounts, but if the proof of the work was to be done, it would be necessary to restore the parts thus omitted.

The question in this last example might be inverted, and it might be proposed to multiply 326 fathoms 3 feet 2 inches, by 25£. 8s. 1d. by saying; if for 1£. I have 326 fathoms 3 feet 2 inches, how many shall I have for 25£. 8s. 1d.? In order to solve this question, I must repeat the first number as many times as the second contains pounds and parts thereof.

MULTIPLICATION.

	326 *fath.*	3 *feet.*	2 *inc.*		
	25£.	8*s.*	1*d.*		
	1630 *fathoms.*				
	652				
3 *feet* -	12 -	3 *feet*	0 *inc.*		
1 - -	4 -	1 -	0		
2 *inc.* -	0 -	4 -	2		
4*s.* - -	65 -	1 -	10		
4 - -	65 -	1 -	10		
1 - -	16 -	1 -	11	6	
1*d.* -	1 -	2 -	1	11	6
	8295 *fath.*	1 *foot.*	0 *inc.*		

This multiplication being done, according to the principles just explained, it is observable, 1st. that the product is different, though the factors are the same; but it is easy to perceive that it proceeds from this; that in the one we calculated for getting pounds, and in the other for fathoms. It is then very important, in complex multiplications, not to change the multiplicand into multiplier, and *vice versa*; that number being always to be taken as multiplicand, which expresses units of the kind sought for, from the state of the question.

It will be observed 2dly, that we have not subdivided the shillings in the multiplier, in the same manner as we had, when this number was the multiplicand; because our object being here to get a number of fathoms, feet and inches, we could not so easily take the 10th of a number of fathoms to reduce it into feet, as it was to reduce pounds into shillings. When the number, expressing pounds and shillings, is the multiplier, the shillings must be divided into aliquot parts of £. or of 20, and that, in the most convenient way, in order to find as few products as possible, and in the speediest and easiest manner.

The proof of a multiplication of complex numbers, may be made by division; but it is more expeditious to do it by a new multiplication, in which you take the half of one of the factors, and double the other, which must evidently give the same product, if both multiplications have been well done.

DIVISION OF COMPOUND NUMBERS.

We must in the first place, observe, that the quotient may be either of the same denomination as the dividend, or of a different denomination; and, secondly, that the dividend being complex, the divisor

may be incomplex, or the dividend being either complex or incomplex, the divisor may be complex; but in order to effectuate the division, (as in all cases the divisor must be reduced to an incomplex number, as we shall see below) the method to be pursued when the dividend and divisor are complex numbers, may equally serve as a rule when the divisor is incomplex; thus, in the examples we are going to examine, the two members of the division will be complex, and we shall distinguish only two cases; the first, when the quotient is of the same denomination as the dividend, and the second when it is of a different denomination; *the question which will give room to a division, will always point out the denomination of the units of the quotient.*

First Case. Question; *suppose 24 fathom 5 feet 8 inches, of a certain work, cost, 214£. 7s. 5d. how much will one fathom cost?*

We know very well, that to resolve this question, we must divide the whole price of the work by the number of fathoms and parts of fathoms which it comprehends, and that the quotient will be of the same denomination as the dividend; but to perform this operation, the divisor must be considered as an abstract number, expressing what part we ought to take of the dividend; for let us remember that to divide a number by 4, for example, or to take the fourth part of a number is the same thing; thus, being asked to divide the number proposed by 24 fathom 5 feet 8 inches, we must know, what part of that number we ought to take, and for that purpose we must reduce 24 fathom 5 feet 8 inches to a number of one denomination, which we may then make use of as an abstract number; to effect this, we may always proceed according to the following method.

Dividend.			**Divisor.**		
£.	*s.*	*d.*	*fath.*	*ft.*	*inc.*
214 ..	7 ..	5	24 ..	5 ..	8
		3			3
643 ..	2 ..	3	74 ..	5 ..	.
		6		6	
3858 ..	13 ..	6	449	.	
3592			£.8 .. 11*s.* .. 10*d.* $\frac{244}{449}$		

266
20
5320
13
5333
449
843
449
394
12
4728
6
4734
449
244

	£.	*s.*	*d.*
	8 ..	11 ..	10
	24*fath.*	5*feet.*	8 *inch.*
	192		
2*s.* - - -	2 ..	8 ..	0
8 - - - - - -	9 ..	12 ..	0
1 - - - - - -	1 ..	4 ..	0
6*d.* - -	0 ..	12 ..	0
3 - -	0 ..	6 ..	0
1 - -	0 ..	2 ..	0
3*feet.* - - -	4 ..	5 ..	11
2 - - - - - -	2 ..	17 ..	$3\frac{1}{3}$
8*inch.* -	0 ..	19 ..	$1\frac{1}{9}$
for remainder,	0 ..	1 ..	$1\frac{5}{9}$
	£.214 ..	7*s* ..	5*d.*

We begin the operation, by causing the units of the lowest denomination to disappear, in multiplying every part of the divisor by the smallest number

which may accomplish the desired object. In this example, 3 is the number; and the multiplication being performed, the divisor is found to be 74 fathom 5 feet, we have multiplied this number by 6 to make the feet disappear, which has given us for product the number 449, which we may now make use of, as an abstract number.

This operation being performed, it is easily perceived, that, since we have made the divisor 18 times greater, we must, (in order not to change the quotient,) render the dividend the same number of times greater; it is what we have done in multiplying it successively by 3 and 6.

This being done, we then proceeded to effectuate the division by operating at first on the pounds, and we found 8 for the quotient, and 266 for the remainder; it is evident, that this remainder being smaller than the divisor, there can be in this quotient no more principal units, that is to say, pounds; but there may be found some shillings; in fact, multiplying this remainder 266£. by 20, to reduce it to shillings, and adding to the product 5320 the 13 shillings of the dividend, we have a new partial dividend, which gives 11 shillings for the quotient, with a remainder of 394 shillings, which, multiplied by 12, to reduce them to pence, give the number 4728; we have added to it the pence of the dividend, and then dividing 4734, we have had 10 pence for the quotient with a remainder, which, put under a fractional form, has completed the quotient.

The proof of the work is placed below the quotient, and we have drawn it here at full length, only to have an opportunity of making some important remarks.

1st. We have not employed for the proof, the divisor which has been made use of in performing the division, but the given divisor, because, suppose we were mistaken in the reduction, it is evident that the

division, though well performed, could only give a false result.

2nd. We have taken for multiplicand the quotient, and not the divisor, because, as the product, which we are in search of, must necessarily be equal to the given dividend, the multiplicand ought consequently to be a number of pounds; this has a reference to the remark which has been already made in the preceding article.

3d. We have not written the fraction, which is at the end of the quotient, because the numerator of this fraction being the remainder of the division, has been added to the product of the quotient by the divisor; but it was necessary first to divide this remainder by 18, which is the number by which we multiplied the given dividend; in effect, it is plainly perceived, that this remainder, resulting from the reduced dividend, ought to be 18 times greater than it otherwise would have been, could we have performed the division by preserving the given dividend.

Second Case. *When the quotient is not of the same denomination as the dividend.* This case can never occur, but when the dividend and divisor are of the same denomination; and then the most simple method is to reduce the dividend and divisor to units of the lowest denomination, which are expressed in the example, and to perform the division as usual, considering the units of the dividend, as if they were of the same denomination, as those required in the quotient.

QUESTION. *It is required to know, how many fathoms of a certain work, we can have performed for* 585£. 8s. 5d. *at the rate of* 54£. 12s. *per fathom.*

It is evident, that we shall have a number of fathoms equal to the number of times that 585£. 8s. 5d. will contain 54£. 12s. that is to say, we must divide the first of these two numbers by the second, and the quotient will express the number of fathoms and parts of fathoms demanded ; but, to effectuate the division, it is to be remarked that the dividend and divisor being of the same kind, we can reduce them to the lowest denomination of this kind, without changing their value, and we can then perform the division as if they were two abstract numbers. The lowest denomination expressed here is pence, we reduce the two terms of the division into pence, by multiplying the number of pounds by 20, which will give shillings for product ; and, by multiplying afterwards all the shillings by 12, to reduce them to pence ; this operation being performed, we shall have

DIVIDEND.	DIVISOR.
140501	13104
13104	10*fath* - 4*ft* - 3*inch* - $\frac{12888}{13104}$.
1st *remainder* 9461	
6	54£. - 12*s*.
	10*fath* - 4*ft* - 3*inch*.
56766	
52416	546£.
	3*ft* - - - 27 - - - 6*s*.
2d *remainder* 4350	1 - - - - 9 - - - 2
12	3*inc* - 2 - - - 5 - - 6*d*.
	remainder, 0 - - - 14 - - 11
52200	
39312	585£. - - 8*s*. - 5*d*.
3d *remainder* 12888	

In following the calculation of the work, we find, that after having obtained 10 in the quotient, the division could not be continued, we then multiplied the remainder 9461 by 6, to reduce it to feet, and in that we acted upon the same principle as in the preceding example; having obtained 4 feet for the quotient, and 4350 for the remainder, we multiplied this remainder by 12, to reduce it to inches.

For the proof, which is here brought forward, below the quotient, we have taken the given divisor for multiplicand, and before we added the remainder of the division to the product, we divided it by 72, because this remainder, not being part of the quotient in the proof, must be considered as a number of pence, resulting from the first remainder of the division 9461*d.* which has been successively multiplied by 6 and by 12.

ALLIGATION.

WE call *Alligation*, the mixture which we make of several commodities of different values, to compose a whole of the same number of parts, equal to one another, and of a mean rate.

Suppose we have, for example, two kinds of wine, one at 15d. and the other at 8d. a bottle, and wish to mix them together; we shall form a mixture, the price of which per bottle, will evidently be the mean rate between 15d. and 8d.

The necessity there is in commerce of making mixtures, gives rise to two kinds of questions, which

form two different cases, which the rule of Alligation shews how to resolve.

First Case. To determine the price of each of the equal parts of the mixture, when we know the value of each of the mixed parts, and the number of the parts of the mixture.

The method to be pursued in this case, is to find the total price of the mixture, divide it by the number of mixed parts, and the quotient gives the price of each of the equal parts of the mixture.

QUESTION. *To form of three kinds of wine, a mixture composed of* 150 *bottles: we have taken* 96 *bottles of wine at* 16*d*, 33 *at* 13*d*, *and* 21 *at* 10*d*. *it is required to find the price of one bottle of this mixture.*

It is evident, that if we knew the total price of the mixture, we should only have to divide it by the number of bottles which compose it, then we should have the price of a bottle of this mixture. Now, to have the whole value of the mixture, we must follow this reasoning. Since one bottle of the first wine costs 16d. the 96 bottles will cost 96 times 16d. that is to say - - - - - - - - - - 1536 pence
the 33 bottles at 13d. each, will be worth 429
the 21 bottles at 10d. each, will be worth 210
therefore the total price of the mixture ——
amounts to - - - - - - - - - 2175 pence.

Dividing this number by that of the bottles, which is 150, it is very plain that the quotient will be the price of a bottle of the mixture, which we find to be $14\frac{1}{2}$d.

Second Case. To determine the parts we are to take, of each commodity to be mixed, when we know

the price of each of the commodities, and the value of the mixture.

First Question. *We wish to compose a wine at* 15*d.* *per bottle, with two sorts of wine, one at* 20*d.* *and the other at* 12*d.* *the bottle, how many are to be taken of each kind of wine?* nothing is easier to be found.

We always dispose the numbers which point out the price of the wines, as we see it done here; we then take the difference between the highest price and the mean rate, that is to say, between 20 and 15*d.* and write this difference, which is 5, on the same line with the lowest price; we afterwards take, in like manner, the difference between the mean rate and the lowest price, that is, between 15 and 12, and we write this difference, on the same line with the highest price; then, if we mix together 3 bottles of wine at 20*d.* and 5 bottles of that at 12d, the mixture will be composed of wine at 15*d.* the bottle.

	20 .. 3
15	
	12 .. 5

For, by each bottle of wine at 20*d.* which we put in to form the mixture, we lose 5*d*, since we are to sell each bottle of the mixture for only 15*d.* then the 3 bottles will make together a loss of 3 times 5d or 15d; but, on the other hand, for each bottle of wine at 12*d.* which we put in to form the mixture, we gain 3*d.* since we are to sell it at 15*d*; thus, for the 5 bottles, we shall gain 5 times 3*d.* or 15*d.* whence it is evident, we gain as much on one side as we lose on the other; therefore, the mixture is such as it ought to be.

We have solved the question generally, without paying attention to the total quantity of the wine we should wish to have in the mixture; but if this quantity was limited, supposing, for example, we had only 30 bottles of wine of the first quality, and the

question was then, to find how many we ought to take of the second, to form with these 30 bottles, a mixture at the price demanded; it is very easily perceived, that we ought to take 5 bottles of wine of the second quality as many times, as we have put in 3 bottles of wine of the first quality; that is, we are to multiply 5 by the number of times, that 3 is comprehended in 30, or by the quotient of 30 divided by 3; thus we are to put in 50 bottles of wine of the second quality.

The better to comprehend the meaning of this method, let us take another example, and suppose we wish to put into the mixture, only two bottles of wine of the first quality; we must then, according to what we have just said, divide 2 by 3, which gives $\frac{2}{3}$, and then, multiplying 5 by $\frac{2}{3}$, we shall have $\frac{10}{3}=3\frac{1}{3}$, which will be the number of bottles of wine of the second quality, which we are to join to the two bottles of wine of the first quality, to form a mixture at the price of 15*d.* the bottle.

There may be occasion for mixing together more than two commodities, as in the following problem.

Second Question. *We wish to make a composition of coffee at* 21*d. the pound, with three sorts of coffee,* 1st *at* 27*d. a pound,* 2nd *at* 25*d. and the* 3d *at* 18*d. how many pounds of each sort must we take?*

We dispose the numbers as in the preceding example, and suppose at first, that we have to mix only two kinds of coffee, one at 27*d.* and the other at 18*d.* the pound; proceeding as in the first question, I write the difference of 27 and 21 by the side of 18, and the difference of 21 and 18 beside 27; it is evident then, that if we take 3 pounds

	27 .. 3	
	25 .. 3	
21		
	18 .. 6 . 4	

of the coffee at 27*d.* and 6 pounds of the coffee at

18*d*. we shall have coffee at 21*d*...Suppose the mixture made; the question, at present, is to bring in the coffee at 25*d*. without augmenting the value of the mixture; to effect this, I proceed in the same manner as above, considering only the two prices 25*d*. and 18*d*, I write, on the same line with 18, and by the side of 6, the difference between 25 and 21, and I also write by the side of 25 the difference between 21 and 18; then I say, that to form the mixture demanded, we must take 3 pounds of the coffee at 27*d*. 3 pounds of the coffee at 25*d*. and 10 pounds of that at 18*d*. for, we shall see, that the loss on one side is equal to the gain on the other; in effect, the 3 pounds at 27d. put in the mixture, will make a loss of 18*d*. and the 3 pounds at 25*d*. will make another loss of 12*d*. which makes the total loss 30*d*. which is precisely equal to the gain, which the 10 pounds of coffee at 18*d*. will produce, since each of them, as we see, gives a gain of 3*d*. It is by following the same method, that we can form mixtures with any number of commodities whatever.

The necessities of commerce give rise to a great variety of questions on mixtures; the principles we have just laid down will suffice to solve them; but we shall, however, here discuss an example which might embarrass the learner, and which is of great use in the coining of money, and in a goldsmith's operations.

Third Question. *A goldsmith has 25 marks of silver, of 12 penny-weights fine, and wishes to make jewels of them of 10 penny-weights fine; what quantity of other matter, of brass for example, must he add to the 25 marks, to accomplish his end?*

The question, therefore, at present, is, to form a mixture with two matters, the value of one of which

may be expressed by 12, the title of the silver, and that of the other by 0, the title of the brass; after this observation, there is no longer any difficulty; we are to employ the same rules we have laid down above, and write the numbers thus;

	12	10
10		
	0	2

According to the solution for 10 marks of silver, we are to put in two of brass; and consequently, with the 25 marks of silver, there must be mixed 5 of brass, which will make in all 30 marks, which, being melted and amalgamated, will produce silver of 10 penny-weights fine.

Fourth Question. *Of which I shall only expose the solution.*

A merchant purchases a quantity of strong brandy at the rate of 25d. the bottle: the market price is only 22d. a bottle; what proportion of water must he put with it, to gain two pence per bottle at the market price.

For 20 bottles of brandy, he must put in 5 of water.

	25	20
20		
	0	5

Note. The question supposes, that, after the mixture, the brandy will still retain the qualities requisite to render it saleable.

RATIOS AND PROPORTIONS.

There are but two ways of comparing one number with another; the one, in considering how much the first surpasses, or is surpassed, by the second; and the second, in considering, how often the first contains or is contained, in the second. The first manner

of comparing two numbers, is called *Arithmetical Ratio*, and the second *Geometrical Ratio*, or simply *Ratio*.

Thus; suppose I have two numbers, such as 12 and 4, and that, in considering how much 12 surpasses 4, I take the difference between these two numbers; 8 the difference is called the *Arithmetical Ratio* between 12 and 4.

But, according to the other manner, if I examine how many times 12 contains 4, or if I take the quotient of 12 divided by 4, three, which is the quotient, is called the *Ratio* of 12 to 4.

Thus each ratio is composed of two numbers, which we call terms; the first term of a ratio is called the *antecedent*, and the second the *consequent*.

A proportion is the equality of two Ratios; whence it follows there are two kinds of proportions, the one Arithmetical and the other Geometrical.

Suppose we have these four numbers, 12, 4, 14, and 6: the Arithmetical ratio, or the difference between 12 and 4, being equal to that between 14 and 6, these four numbers form an Arithmetical Proportion, which we write thus; 12..4::14..6; and which is pronounced in this manner, 12 is to 4, as 14 is to 6. This kind of proportion being of no use in commerce, we shall content ourselves with having indicated it, and we shall pass immediately to Geometrical Proportions, which we simply call proportions.

Let there be the four numbers, 12, 4, 21, and 7; the ratio of 12 to 4 being equal to that of 21 to 7, the four numbers form a proportion, which is thus written; 12 : 4::21 : 7, and which is pronounced, 12 is to 4 as 21 is to 7.

A proportion, therefore, is composed of four terms, the first and third are called the *Antecedents*; the second and fourth, the *consequents*; the first and fourth are denominated the *extremes* of the proportion; the second and third, are called the *means*.

The essential property of every Geometrical proportion, and the only one of which we shall speak here, is, that *the product of the extremes, is equal to the product of the means.* Suppose we have the proportion 12 : 4 :: 21 : 7; I say, that $12\times7=4\times21$; for the ratio of 12 to 4, which is the same thing as the quotient of 12 divided by 4, is equal, (since there is a proportion) to the ratio of 21 to 7, or to the quotient of 21 divided by 7; thus, from the equality of the ratios, we have $\frac{12}{4}$ equal to $\frac{21}{7}$; let us reduce these fractions to the same denominator, whereby we shall not change their value, and we shall still have $\frac{12\times7}{4\times7}$ equal to $\frac{21\times4}{4\times7}$; but since these two fractions, always equal, have the same denominator, it necessarily follows that their numerators are equal; therefore 12×7, the product of the extremes, is equal to 21×4, the product of the means. As in this reasoning the numbers of the proportion, are not particularly considered, we perceive very well, that this reasoning is general, and that demonstration, which we have just given, is applicable to every Geometrical proportion.

From this property results a very important consequence, it is, that, *knowing three terms of a proportion, we can always find the fourth.* The unknown term may be an extreme or a mean.

If it be an extreme, it is equal to the product of the two means, divided by the other extreme; for, if I have the proportion, 3 : 9 :: 5 : x (I call x the un-

known term ;) since there is proportion, I shall have the product of the extremes equal to that of the means, which gives $3\times x=9\times 5=45$, but since x taken three times, or three times x, is equal to 45, it follows, that x alone is equal to the third of 45, or $x=\frac{9\times 5}{3}$ therefore, &c.

If it be a mean, it is equal to the product of the two extremes, divided by the other mean ; for, suppose the proportion, 9 : 3 : : x : 5 ; making the product of the means equal to that of the extremes, we shall have $3\times x=9\times 5$, and consequently $x=\frac{9\times 5}{3}$.

It is necessary, in this place, to make an observation, from which we may often derive the greatest advantages to simplify the calculation of proportions ; it is, that we can always multiply or divide two terms of a proportion by a same number, without destroying the proportion, provided we operate at the same time upon one extreme and one mean ; it is easily conceived, that the product of the extremes, always remaining equal to that of the means, there is still proportion ; but to demonstrate this truth in an evident manner, let us descend to some detail.

I say, in the first place, that we can divide the two first terms of a proportion, or the two second ones, by a same number, without destroying the proportion ; suppose we have the proportion, 150 : 50 : : 24 : 8 ; dividing the two first terms by 10, and the two last by 8, we have to prove that the quotients will also be in proportion, that is, that we shall have 15 : 5 : : 3 : 1 ; since the four given terms are in proportion, we shall have the ratio of 150 to 50, equal that of 24 to 8, that is $\frac{150}{50}=\frac{24}{8}$, but as we can always divide

the two terms of a fraction by a same number, without changing their value, if we divide the two terms of the first by 10, and the two terms of the second by 8, we shall then have $\frac{15}{5}=\frac{3}{1}$, and these two ratios being equal, will then give the proportion 15 : 5 :: 3 : 1.

I say secondly, that we can divide the two antecedents of a proportion, or the two consequents by a same number, without destroying the proportion. Suppose we have the proportion 24 : 96 :: 9 : 36, dividing the two antecedents by 3, and the two consequents by 4, we have to prove that the quotients will also be in proportion, that is to say, that we shall have 8 : 24 :: 3 : 9. The above proportion gives $\frac{24}{96}=\frac{9}{36}$, but as it is plain, that two equal quantities may be divided by a same quantity, without changing the equality; therefore, if we divide these two equal fractions by 3, we shall have $\frac{8}{96}=\frac{3}{36}$; but as it is not less true that we can multiply each of these last fractions by 4 without destroying the equality, and that to multiply a fraction by 4, or to take the fourth of its denominator, is the same thing, we shall then have $\frac{8}{24}=\frac{3}{9}$; and these two equal ratios will give the proportion demanded 8 : 24 :: 3 : 9

THE RULE OF THREE.

The *Rule of Three* has for its object, to find out the 4th term of a proportion, three of which are given by the state of the question.

It may be either *Simple* or *Compound*; it is simple when the question contains but three known quantities; and it is compound when it contains more.

Arithmeticians divide it also into *Direct and Inverse*;

but this distinction is unnecessary, embarrassing to beginners, and sometimes even difficult to be understood by practitioners. The rule of three, either indirect, simple, or compound, may always be solved by the rule of three direct compound; founded on th.s observation, that every question which gives rise to a rule of three, can always be considered as containing *two causes* and *two effects*; but the effects being necessarily and *directly* proportional to the causes, the rule will then be always *direct*, and the proportion will be thus established; *the* 1st *cause is to its effect, as the* 2nd *cause is to its effect.* The only difficulty in this rule, is, to distinguish the causes and the effects, which will easily be learned by the help of a few examples.

Simple Rule of Three.

1st Question. *If* 27 *ells of cloth cost* 500£. *how much will* 48 *ells of the same cloth cost?*

It is obvious that the ells of cloth are the causes of the money expended, therefore the proportion is thus established.

1st cause.	1st effect.	2d cause.	2d effect.
ells.	£.	*ells.*	
27 :	500 ::	48 :	X whence we draw the extreme

$$X = \frac{\overset{\pounds.}{500 \times 48}}{27} = \overset{\pounds.}{888} - \overset{s.}{17} - \overset{d.}{9\tfrac{1}{4}}$$ *the price of the* 48 *ells.*

Second Question. *If* 24 *fathoms* 3 *feet* 6 *inches, cost* 215£. 8*s.* 4*d. how many fathoms shall we have for* 427£. 10*s.* 6*d?*

In this question, the fathoms are the causes, and the sums expended the effects; thus we shall have this proportion.

1st cause.	1st effect.	2d cause.	2d effect.
fath. ft. inc.	*£. s. d.*	*fath.*	*£. s. d.*
24 - 3 - 6 :	215 - 8 - 4 ::	X :	427 - 10 - 6

The most simple method of treating proportions, when there are complex numbers, is to change two of these terms, which are necessarily of the same kind, into incomplex numbers, by reducing them to units of the lowest denomination expressed in the two numbers. We shall, therefore, reduce the two consequents of the proportion, into pence; but, before we do that, we can simplify the calculation, by taking the half of the two first terms, and after that, the half of the two consequents, so that the proportion will be,

fath. ft. inc. 12 - - 1 - - 9 : *d.* 12925 :: *fath.* X : *d.* 51303 ; whence we draw

$$X = \frac{(12 \text{ - - } 1 \text{ - - } 9) \times 51303}{12925} = 48 - 4 - 8\tfrac{10555}{12925}$$

(*fath. ft. inc.* above the first term; *fath. ft inch.* above the result) *the number of fathoms demanded.*

Rule of Three Compound.

The method to be followed in this case, is to reduce the question to a simple Rule of Three; which is easily done, by multiplying together the numbers which form a same cause, and by multiplying together in like manner, the numbers which compose the same effect.

1st Question. *If* 3 *workmen, labouring during* 5 *days, at the rate of* 8 *hours a day, dig* 45 *fathoms of a ditch* 2 *feet wide, how many fathoms of a ditch* 3 *feet wide, will be dug by* 7 *labourers, working* 9 *days, at the rate of* 10 *hours per day.*

Let us begin by distinguishing the causes and the effects. It is evident, the 3 labourers, the 5 days and 8 hours a day, which they worked, are the 1st cause; and that the 45 fathoms of a ditch, 2 feet wide, are the

1st effect; it is also plain, that the 7 labourers, the 9 days and the 10 hours per day, which they are to work, are the 2d cause; and that the number of fathoms, which I call X, of a ditch 3 feet wide, is the 2nd effect; thus then we can dispose the numbers in columns, distinguishing the columns by causes and effects.

1st. cause.	*1st. effect.*	*2nd cause.*	*2nd. effect.*
3 *workmen*	45 *fathom*	7 *workmen*	X *fathom*
5 *days*	2 *feet*	9 *days*	3 *feet.*
8 *hours*		10 *hours*	

The proportion, then, naturally presents itself; it exists between the causes and the effects; but it now remains to reduce each cause and each effect to one number, in order to bring this case to a simple rule of three; to effect which, we have only to multiply the numbers of each column by one another; in fact, it is evident, that 3 labourers, working 5 days, perform the same work as 5 times 3 labourers, or 15 labourers, working one day; and that 15 workmen labouring 8 hours a day, perform the same work as 8 times 15 workmen, or 120 workmen labouring one hour per day; we likewise see, that 45 fathoms of a ditch 2 feet wide, occasion the same work as twice 45 fathoms, or 90 fathoms of a ditch one foot wide.

Reasoning in the same manner, we shall see, that the 7 labourers, the 9 days, and the 10 hours work per day, are equal to 630 labourers working one hour a day; and that the number X fathoms of a ditch, 3 feet wide, is the same thing as 3 times X fathoms of a ditch one foot wide; thus, then, comparing the causes and the effects, we shall have the proportion $3 \times 5 \times 8 : 45 \times 2 :: 7 \times 9 \times 10 : X \times 3$.

It is proper to form the proportion by only indicating the calculation, to give a facility of making the reductions, which may often take place between the several terms; here, for example, we see, that the

two first terms of the proportion, may both be divided by 3, 5 and 2, and that the two last may be divided by 3; then the proportion becomes $1 \times 1 \times 4\,lab : 3 \times 1\,fath :: 7 \times 3 \times 10\,lab : X fath.$ and dividing again each of the antecedents by 2, and suppressing 1, which never multiplies, we shall have $2 lab : 3 fath :: 7 \times 3 \times 5 lab : X :$ or $2 lab : 3 fath :: 105 lab : X fath.$; a proportion very much simplified, giving

$$X = \frac{105 \times 3 \;\; lab. \; fath.}{2 \; lab.} \text{ which is the same thing as } \frac{105 \; lab.}{2 \; lab.} \times 3 \; fath.$$

therefore, to have the value of X, we must multiply 3 *fath.* by the quotient of $\frac{105 \; lab.}{2 \; lab.}$ *which is an abstract number*,* the operation being performed, we have $X = 157\,fath.$ $3\,feet.$ which is the number of fathoms demanded.

Second Question. *An officer sets out with a troop of soldiers, to occupy a fortress, with orders to remain there 16 days; he carries with him money necessary for the pay of the soldiers, supposed to be 7d. a day for each man; but on his route, he receives new orders, by which he is enjoined to remain in the fortress 5 days longer. What was then to be the pay of each soldier per day?*

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* We have made this remark, to show, that in a proportion which contains no complex numbers, we may treat all its terms as abstract numbers; but when we obtain the final value of X, we may then consider the result as expressing the kind of units, which the state of the question requires; here the kind of units which we sought, were fathoms, thus having $X = \frac{105 \times 3}{2}$ we might have looked upon the numerator of this fraction as a number of fathoms to be divided by 2.

This question gives rise to an indirect simple Rule of Three; but we shall show that it can be solved by the Rule of Three Compound, which must always be done by considering in the question two causes and two effects. Here the two effects are the same, (this unity of effects always takes place in the Rule of Three called Inverse), it is a sum of money destined to pay the soldiers, we may express it by 1; this sum is to be expended on one side by giving 7d. to each soldier during 16 days, and on the other by giving X pence to the soldiers during 21 days; thus, in arranging the numbers by causes and effects, and forming the proportion immediately, we shall have $7 \times 16 : 1 :: 21 \times X : 1$; dividing the two antecedents by 7, which gives $16 : 1 :: 3X : 1$, we have $X = \frac{16}{3}d. = 5\frac{1}{3}d.$ the sum which the captain is to give to each soldier per day, being to remain five days longer in the fortress.

Third Question. *How many ells of cloth $\frac{2}{3}$ wide, are sufficient to make a coat of equal extent with one, which hath in it $1\frac{1}{2}$ ells of cloth, $\frac{5}{4}$ wide?*

This question, which would give rise to a rule of three simple indirect, may be solved by the rule of three direct compound. The two causes here, are the ells of cloth, with their respective breadths, and the effect, which is the same for the two causes, is the coat; thus we shall have the proportion $(1 + \frac{1}{2}) \times \frac{5}{4} : 1 :: x \times \frac{2}{3} : 1$.

To manage, in a convenient and general manner, the proportions, in which there are found fractional terms; we must begin by making the denominators disappear, which can always be executed with great ease, by transferring the denominator of a term which is an antecedent, as a multiplier to its consequent, and *vice versa*; but, before we make this operation, it is necessary to reduce all the numbers of the same term

to the same denominations; thus the above proportion will be at first $\frac{3}{2} \times \frac{5}{4} : 1 :: x \times \frac{2}{3} : 1$; then, making the denominators disappear, we shall have $3 \times 5 : 8 :: x \times 2 : 3$, whence $X = \frac{3 \times 5 \times 3}{8 \times 2} = \frac{45 \text{ ells}}{16} = 2\frac{13}{16} \text{ ells}$; which is the number of ells demanded.

These examples are sufficient to show the spirit of the rule, and to convince us, that *every question which gives rise to the Rule of Three indirect or inverse, may always be solved by the direct compound rule of three.*

Proof of the Rule of Three.

To be certain that the rule of three has been well performed, it is necessary to put, instead of x, its value in the proportion established, before any reduction be made, and then make the reductions by as different ways as possible, in order to avoid the faults into which we might have fallen; and, if we find the produce of the extremes, equal to that of the means, we may conclude that the value of x is right, and that the sum has been well performed.

RULE OF FELLOWSHIP.

The intent of this rule, is principally to divide the gain or loss in trade, proportionably to the respective stocks of the partners.

This rule may be either simple or compound. It is *simple*, when all the stocks of the partners are in

trade during the same length of time; it is *compound*, and called *Fellowship with time*, when the stock of the partners are not in trade during the same length of time; but to solve the question in this case, we must always reduce it to one of simple fellowship. The method to be pursued in order to solve a question in fellowship, is founded on this evident proportion, *As the whole stock is to the whole gain, so is the particular stock to its particular gain.* We are going to make an application of this in the following questions.

SIMPLE FELLOWSHIP.

QUESTION. *Three merchants formed a company and gained* 4000£.

the first put in - - -	2000£.
the second - - - -	5844
the third - - - -	7548
total stock	15392£;

it is desired to know each man's share of the profit, in proportion to the sum he put in.

Making use of this proportion; *as the whole stock is to the whole gain, so is the particular stock to its gain*, it is very easy to solve the question; for we shall have these three proportions, which will give the share of each one.

whole stock.	whole gain.	stock of 1st.	
15392£ :	4000£ ::	2000£ :	to the gain of 1st=519£. 15s. 0d. $\frac{60}{481}$
15392 :	4000 ::	5844 :	to the gain of 2d=1518 - 14 - 2 $\frac{310}{481}$
15392 :	4000 ::	7548 :	to the gain of 3d=1961 - 10 - 9 $\frac{111}{481}$

Proof, - 4000£. 0 - 0 - 0

The proof of this rule is made, by adding together all the particular gains, the sum of which must be equal to the whole gain.

REMARK. It is necessary to observe, that the first ratio of each of these proportions being the same, it is very advantageous, in order to abridge the calculation, to reduce this ratio to its simplest expression, and to effect that, we may make use of the method taught, for reduction, by way of trial.

Here this reduction, being made, we have; first proportion; 481 : 125 :: 200 : to the gain of the 1st,=&c.

COMPOUND FELLOWSHIP;

OR FELLOWSHIP WITH TIME.

First Question. *Three merchants have put in partnership; the* 1st 2000£. *which remained six months in trade; the 2nd* 3454£. *which were in* 8 *months; the 3d* 5482£. *which remained in one year; the profit resulting from these three stocks, amounted to* 4000£. *what was the share of each one in proportion to his stock, and to the time that each stock remained in trade?*

To solve this question, we must reduce it to one of Simple Fellowship, which can always be done, by bringing each stock to a same unit of time in trade. Then we shall take a month for the unit of time, and we shall reduce all the stocks to it, by pursuing this reasoning.

The 2000£. the first man's stock, which remained six months in trade, must produce the same profit, that 6 times 2000£. or 12000£. remaining in for one month, would produce. In like manner,

The 3454£. the second man's stock which remained 8 months in trade, must produce the same profit, that 8 times 3454£. or 27632£. remaining in for one month, would produce:—In fine,

The 5482£. the 3d man's stock, which remained 12 months in trade, must produce the same profit that 12 times 5482£. or 65784£. remaining in one month, would produce; thus the three stocks are reduced to a unit of time, and now the question may be expressed in this manner;

Three merchants formed a company for one month, and they gained 4000£.

The first put in	- -	12000£.
The second	- - -	27632
The third	- - - -	65784
	Total stock	105416

what share of the profit has each man?

This now becomes a question of simple fellowship, which is solved by the three following proportions.

£.	£.	£.		£. s. d.
105416 :	4000 ::	12000 :	to the gain of the 1st=	455 - 6 - $9\frac{4263}{13177}$
105416 :	4000 ::	27632 :	to the gain of the 2d=	1048 - 9 - $10\frac{6074}{13177}$
105416 :	4000 ::	65784 :	to the gain of the 3d=	2496 - 3 - $4\frac{2840}{13177}$
			Proof, -	4000 - 0 - 0 - - 0

NOTE. The Ratio of the whole stock, to the whole gain, may be reduced to that of 13177 : 500; which simplifies the calculation in each proportion.

REMARK. Though the sum of the particular gains be equal to the whole gain, yet the division of

the gains may be false; because some error of calculation may have been made in reducing the stocks to a unit of time; thus, then, it is very necessary, before we establish the proportions, to be certain, that the calculation of the reduction of the stocks, to a unit of time, is right.

Second Question. *Three merchants formed a company, and gained in two years, the sum of* 1755£. *the* 1st *put in* 8000£. *which remained all the time in trade; the* 2nd *put in at first* 7000£. *but* 9 *months after, he put in* 3000£. *more: the* 3d *put in at first* 10000£. *but a year after he withdrew the half of it; it is required to tell eaeh man's share of the whole gain, proportionably to his stock, and to the time it remained in trade?*

We must first reduce all the stocks to a unit of time, and here again we shall take a month for the unit. Though this question is more complicated than the preceding, on account of the changes, which the two last merchants made in their stocks, it is nevertheless easily solved by the same method, by only considering the different stocks of each of the merchants, one after another, and from one period of time to another. Let us enter into details.

The first merchant, who put in 8000£. which remained in trade all the time, has the same right to the whole profit, as if he had put in 24 times 8000£. or 192000£. for one month.

The second merchant, having at first put in 7000£. which remained in trade 9 months, and at that period of time, having put in 3000£. more, making then a sum of 10000£. which are found to have remained 15 months in trade, has the same right to the whole gain, as if he had put in for one month, on one

side 9 times 7000£. or 63000£. and on the other, 15 times 10000£. or 150000£. which makes in all a sum of 213000£.

The third merchant, having put in at first 10000£. which remained in trade 12 months, and at that time having withdrawn the half of his fund, so that his stock was then only 5000£. which remained 12 months in trade, has the same claim to the whole profit, as if he had put in for one month, on one side 12 times 10000£. or 120000£. and on the other 12 times 5000£. or 60000£. which amount in all to the sum of 180000£.

The question, therefore, is now reduced to one of simple fellowship, and may be thus expressed.

Three merchants formed a company for one month and gained 1755£.

The first put in	-	192000£.
The second	- - -	213000
The third	- - -	180000
whole stock		585000

; *what is each man's share of the gain?* we obtain the shares by means of the three following proportions.

£.	£.	£.		£.
585000 :	1755 ::	192000 :	to the gain of the 1st	=576
585000 :	1755 ::	213000 :	to the gain of the 2nd	=639
585000 :	1755 ::	180000 :	to the gain of the 3d	=540
			Proof, -	1755

NOTE. The first Ratio of each of these proportions, may be reduced to that of 1000 : 3 ; and then taking away three cyphers from the right hand of

each of the antecedents, all these proportions are much simplified, and the first becomes 1 : 3 : : 192 : to the gain of the 1st, = &c.

REMARK. When we have a great number of proportions, which commonly happens in the division of shares in a bankruptcy, if the two first terms after reducing them to their simplest expression, were still considerable, we could, to facilitate the calculation in the proportions, form a table of the products of each of these numbers, which would very much abridge the work.

We call a table of the products of a number, the different products of this number, by 1, 2, 3, 4, 5, 6, 7, 8, 9. To form this table, we write all the different products below each other on a loose sheet, which we have under our eyes, when we employ the number in question, as a multiplicand, or as a divisor; we also write at the left hand of each of the products of the table, the figure which has served as a multiplier.

RULE OF INTEREST.

By *Interest*, we mean the money due for a sum lent out, which is called the *Principal*. The chief object of the rule of interest, is, to determine the money due according to the conditions of the loan.

Interest may be either *simple* or *compound* : It is *simple*, when it proceeds only from the principal. It is *compound*, when to the interest of the principal, are superadded the *interests upon interests*.

The *rate* of Interest is fixed at so much *per-cent per-annum*, as at 3, 4, 5, 6, &c. *per-cent*; which means that for 100£. lent, an interest is demanded *per-annum*, of 3, 4, 5, 6, &c.

SIMPLE INTEREST.

We must distinguish two cases: the first, when the loan is for one year; the second, when it is for more or less than one year: but in this 2nd case, the question may be begun to be solved as in the first, and then by the multiplication of complex numbers, it will be easy to come up to the result demanded.

First Case; the rule is solved by this proportion, if 100£. gain so much interest per-annum, how much will the sum lent gain during the same time.

QUESTION. *It is required to tell the interest of 7450£. for one year, at 4 per-cent per-annum.*

We shall say; if 100£. gain 4£. how much will 7450£. gain? and then we shall have 100£.4£:: 7450£: to the interest required, =298£.

Second Case, Question. *What is the interest of 8474£. 8s. 4d. for 7 years 10 months and 18 days, at 5 per-cent per-annum?*

To solve this question, we must first find the interest of 8474£. 8s. 4d for one year; we find it by this proportion, 100£: 5£:: 8474£. 8s. 4d: x; and dividing the two first terms by 5, we have 20 : 1:: 8474£. 8s. 4d: x=423£. 14s. 5d. the interest for one year; now multiplying this sum by the number of years and parts of years, (being the time that the

loan remained out at interest) the product will evidently give the solution of the question.

	£.	s.	d.	
	423	14	5	
	7 *years.*	10 *months.*	18 *days.*	
for 7 *years*	2966	0	11	
6 *months*	211	17	2	$\frac{1}{2}$
3	105	18	7	$\frac{1}{4}$
1	35	6	2	$\frac{5}{12}$
*15 *days*	17	13	1	$\frac{5}{24}$
3	3	10	7	$\frac{53}{120}$
Interest required,	3340£.	6s.	7d.	$\frac{49}{60}$

REMARK. When the loan is in dollars, and the interest is at 6 per-cent, as the law rates it in this country, the simplest method then to be pursued, is, to find at first the interest of the sum for two months, which is obtained by separating with a dash at the right hand of the number, the two first figures, which we count as *cents*, and the other figures of the same number express dollars. This method is easily comprehended, since the interest for a year at 6 per-cent, is reduced to 1 per-cent for 2 months, that is to say, to the hundredth part of the loan. For example, the interest of 3457 dollars, for two months, is 34 dollars and 57 cents.

* The month is supposed to consist of 30 days.

Third Question. *A man pays his creditor* 500£, *both for principal and interest, at* 6 *per-cent, for* 10 *years. What was the principal?*

This question differs from the preceding, but it may be solved by an almost similar method, by saying, since the interest of 100£. is 6£. *per annum*, the interest of 100£. for 10 years will be 60£. thus the interest and the principal of 100£. for 10 years, will be 160£. then it is evident that we have this proportion; 160£. the principal and interest of a hundred pounds : 100£ : : 500£. the principal and interest : to the principal required, which we find =312£. 10*s*.

We can vary questions of this nature very much, but we shall always solve them with ease, by the same principles just employed.

COMPOUND INTEREST.

As to questions of Compound Interest, we can resolve them by means of the rules above, by taking each year successively, the interest of the sum due at the end of the preceding year.

Algebra directs us to a very easy method of solving questions of *Compound Interest*.

RULE OF DISCOUNT.

Discount is an allowance made upon a sum of money, the payment of which is demanded before it becomes due.

The Rule of Discount teaches us to find the amount of the allowance, when we know the rate of discount, which ought to be the same as the interest fixed by the law. The Rule of Discount amounts to this question of interest; find the interest of a principal, which, joined with this same interest, forms a given sum. In effect, let us suppose a bill of exchange of 3000£. at 6 months sight; since the merchant who has in hand this bill of exchange, is to pay it only at the end of 6 months, he is in the same state as if a certain sum had been lent to him, which, joined to its interest, amounts to 3000£; and as if the terms of the loan were such, that he was to remit the sum and its interest, at the expiration of 6 months; but if the proprietor of the bill of exchange requires to be paid before the time has expired, the merchant then ought only to remit him the sum supposed to be lent, and to retain for himself the surplus, which we call *Discount*. The method we are to follow, to find the discount, is general; we must always begin by seeking the interest of 100£. for the time that the bill of exchange was to run; then adding this interest to the 100£. we shall make this proportion; the principal and interest of 100£. is to the interest, as the amount of the bill of exchange, is to the discount required.

Question. *What is the discount of a bill of exchange of* 6484£. *for* 4 *months and* 18 *days, at* 6 *per cent?*

We must first seek the interest of 100£. for 4 months and 18 days; we find it by reasoning thus; if in 12 months we have 6 pounds interest, how many shall we have in 4*m.* 18 *days*? that is to say, 12*m* : 6£ :: 4*m.*.18*d* : X; this proportion becomes, by dividing the two terms of the first ratio by 6, and afterwards taking the half of the two antecedents,

$1m : 1£ :: 2m.\ 9d : X = 2£\frac{3}{10}$;* the interest of 100£. for 4m. 18d.

Now to find the discount, we shall make this proportion $102+\frac{3}{10} : 2+\frac{3}{10} :: 6484 : X$; making the denominators disappear, we shall have $1023 : 23 :: 6484 : X = 145£. - 15s. - 6d.\frac{334}{341}$, which is the discount demanded. Subtracting the discount from the amount of the bill of exchange, the difference, which is $6338£. - 4s. - 5d.\frac{7}{341}$, will be the sum due to the owner of the bill of exchange.

For the proof of the rule we must make this proportion, $100 : 2\frac{3}{10} :: 6338£.-4s.-5d.\frac{7}{341} : 145£.-15s.-6d.\frac{334}{341}$, which will give the product of the extremes equal to that of the means, if the operation has been well performed.

RULE OF FALSE POSITION.

The *Rule of False Position*, consists in dividing a number into parts proportional to numbers, which we determine relatively to the state of the question.

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* Here, beginners are apt to be puzzled; they know not what to conclude from this proportion, considering that 1 neither multiplies nor divides; but they must take notice, that X, being, by the state of the question, a number of pounds, and equal (as the 4th number of the proportion) to $\frac{1£. \times (2m.\ 9d.)}{1m.}$ they must, in this expression, take for multiplicand 1£. and for multiplier 2 months 9 days, which, according to the rules of Complex Multiplication, gives $X = 2£.\frac{3}{10}$.

We cannot better explain this rule, than by examples.

Question First. *Divide* 658£. *among three persons, in such a manner, that the second may have three times as much as the* 1*st, and the third as much as the other two together.*

We plainly see, that if we knew the part of the first, we should easily obtain that of the two others; let us suppose this part to be 1£. then the part of the second will be 3£.and that of the third 1£+3£=4£; the whole sum of these three parts will be 8£; thus the supposition is false, for the whole amount of the parts required, ought to be 658; nevertheless, as the supposed parts are evidently proportional to the true parts, this false supposition will serve us to solve the question; for, we may say, the whole of the false parts is to the first false part, as the whole of the true parts is to the first true part. Thus we have 8£:1:: 658£: the first true part - - - =82£. 5*s*.

Taking three times this part, we shall have the 2d true part - - -	=246	15
Adding these two 1st parts, we shall have the 3d true part - - -	=329	0
Proof - - -	658	0

For the proof, it is evident, that the whole sum of the parts must be equal to the number to be divided.

Question Second. *Find a number, whereof the half, the fourth, and the fifth, make together* 60.

Let us represent the number sought by 1; its half will be $\frac{1}{2}$, its fourth $\frac{1}{4}$, its fifth $\frac{1}{5}$; I add together these three fractions, which give me the fraction $\frac{19}{20}$ for the whole sum of the supposed parts; the supposition is false, but it will give the true number sought, by making this proportion; if the fraction $\frac{19}{20}$ con-

tains the half, the fourth, and the fifth of 1, of what number does 60 contain the half, the fourth, and the fifth parts? Thus we have $\frac{19}{20}$: 1 :: 60 : the number sought $= 63\frac{3}{19}$.

Question Third. *Divide the number 720 into 3 parts, in such a manner, that the 1st may be to the 2d. as 3 is to 4; and the 2d to the 3d, as 5 is to 6.*

If we knew the first part, we should have the others; let us suppose it to be 1; then making this proportion 3 : 4 :: 1 : a fourth term; this fourth term $=\frac{4}{3}$ will evidently represent the second part; and then making this other proportion 5 : 6 :: $\frac{4}{3}$: a fourth term, this fourth term $=\frac{8}{5}$ will represent the third part; the whole sum of these three parts is $1+\frac{4}{3}+\frac{8}{5}$, which added together, make $\frac{59}{15}$; whence we see that the supposition is false; but we shall have the first true part by making this proportion, $\frac{59}{15}$: 1 :: 720 : the first true part - - - - - - - - - - - - - $183\frac{3}{59}$

Now, according to the state of the question, we have 3 : 4 :: $183\frac{3}{59}$: the second true part - - - - - - - - - - $244\frac{4}{59}$

5 : 6 :: $244\frac{4}{59}$: the third true part - - $292\frac{52}{59}$

Proof - - - - 720

Question Fourth. *Divide 300£. among 3 persons, in such a manner, that the 2d may have twice as much as the 1st, and 6£. more; and the 3d as much as both the others, and 10£. more.*

We must observe here, that since the second person is allowed 6£. and the 3d 16£. whatever may be the part of the 1st, we plainly see, that we must first subtract these two numbers from the sum to be

divided, which reduces it to 278£.; then the question becomes just like the first example; now the question is to divide 278£. among 3 persons, in such a manner, that the 2d may have twice as much as the 1st, and the 3d as much as both the others together.

Thus, if we suppose 1£. to be the part of the 1st. 2£. will be the part of the 2d, and 3£. that of the 3d; the whole sum of the false parts will then be 6£. which gives the proportion.

£. £.	£.	s.	d.
6 : 1 :: 278 : the first true part - -	46	6	8
Doubling this 1st part. and adding 6£, we have the 2d part - - - -	93	13	4
Adding to the sum of these two parts, 10£. we have the 3d part - -	155	0	0
Proof - - -	300	0	0

Question Fifth. *Divide* 300£. *among* 3 *persons, in such a manner, that the* 2d *may have twice as much as the* 1*st, less* 6£. *and the* 3d *as much as the two others, less* 10£.

Since, in the preceding example, we were obliged to subtract 22£. from the sum to be divided, it is plain that we must, on the contrary, add them in this instance, and to find the first true part, make this proportion.

	£.	s.	d.
6 : 1 :: 322 : the first true part - -	53	13	4
Doubling this 1st part, and subtracting 6£. we have the 3d part - -	101	6	8
Subtracting 10£. from the sum of these two parts, we have the 5th part	145	0	0
Proof - -	300	0	0

THE FORMATION OF POWERS, AND EXTRACTION OF ROOTS.

We call the *power* of a number, the products which we find by multiplying it by itself a certain number of times; and we call the *root* of the *power*, the number, which, being multiplied by itself a certain number of times, produces this *power*.

After this definition, we see very well, there is no difficulty in finding the *power* required of any number whatever.

Every number is itself its first *power*; the 2nd. *power*, or the *square* of a number, is the product of that number, multiplied by itself; the 3d *power*, or the *cube* of a number, is the product of that number by its square, the 4th *power* of a number is &c. &c.

We must remark, that all the *powers* of 1 are 1; for 1 multiplied by itself, as often as we please, can give but 1. If there is no difficulty in finding any *power* whatever of a number, there is none, also, in obtaining the *root* of a number, considered as a *power*, we call the *square root*, the root of the second *power*, the *cube root*, that of the third, the fourth root that of &c. and so on.

We shall here only lay down the method of extracting the *square* and *cube roots*, as being the only ones of which, we may have *any need in Arithmetic*.

EXTRACTION OF THE SQUARE ROOT.

The method employed to *extract* the *square root* of a number, is founded on the knowledge we have of

the formation of a square. But we must first observe, that every square which is composed but of two figures, can have only one figure for its root, and then, by means of the multiplication table, it will always be easy to know the square root of a number composed of two figures, or of the greatest square which shall be therein contained. Thus, the question is reduced, to find the root of a square, composed of more than two figures.

I say, first, that this square must have tens and units for its root; for 100 which is the least number of three figures, has for its root 10, which is composed of two figures. Let us see then, what a square is composed of, which has tens and units for its root.

Let us take 24 for the root, and form of it the square, taking care to write separately each of the partial products.

$$\begin{array}{r} 24 \\ 24 \\ \hline 16 \\ 80 \\ 80 \\ 400 \\ \hline 576 \end{array}$$

I have then 576 for the square of 24; I see that this number is composed of 400, *which is the square of the tens*, and of twice 80 or 160, *which is the double of the tens by the units; and lastly of the square of the units*. With this knowledge, we can extract the square root of any number whatever.

Let it be proposed, for example, to extract the root of - - - - - - - - - - 6 | 20 | 54 | 19

```
6 | 20 | 54 | 19 | 2491
4                 |------
-----             | 44
22,0              | 489
176               | 4981
-------
 445,4
 440 1
 -------
   5319
   4981
   -----
    338
```

I remark at first, that this number, being composed of more than two figures, has necessarily tens and units for its root; then it is composed *of the square of the tens*, *of the double of the tens by the units*, and *of the square of the units*; but the square of the tens being a number of hundreds, has two places to its right; if, then, we separate the two first figures of this number towards the right, it is certain that the part which is on the left, will contain the square of the tens.

But I remark, that this number, being itself composed of more than two figures, has still tens and units for its root;* thus, also, it contains *the square of the tens*, *the double product of the tens by the units*, and *the square of the units*, but the square of the tens is a number of hundreds, which has two places at its right; if then I separate the two first figures to the right of this number; the part on the left will contain the square of the tens; but this number, being itself still composed of more than two figures, contains tens and units for its root, therefore we must still separate two

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* In a number, any figure whatever, which exprefsed tens in comparifon to the figure which is on its right, may be confidered as a number of units in comparison to the figure which is on its left.

figures on the right; then there remains only one figure, which must contain the square of the tens.

In effect, this square is contained therein, but with an excess which results from the double product of the tens by the units. I then take the root of the greatest square comprised in 6, this greatest square is 4, and its root 2, which I write, as we see, to the left of the number, as if it were a divisor; now I square the tens of the root, which gives me 4, which I subtract from 6, and to the right of the remainder 2, I bring down the next period.

I remark, that the number 220, from which I subtract the square of the tens, contains no more than *the double product of the tens by the units, and the square of the units*; but the double product of the tens by the units, is a number of tens, which has one place to the right; then if I separate one figure to the right of this number, the part on the left will contain the double product of the tens by the units; if, therefore, I divide this number by the double of the tens, I shall have the units in the quotient; the double of the tens is 4, which I write below the root, and divide 22 by 4; the quotient is 5, but I try it before I write it; and this is the method.

Since 220 is composed of the double product of the tens by the units, and of the square of the units, if I take the square of 5, which I suppose to be the units, and add it to the product of 4 by 5, which will be the double of the tens by the units, I shall have a sum, which ought to admit of being subtracted from 220; I see that the subtraction is not possible; thence I conclude, that 5 is too great; I try 4, which is found to be the right figure, I then write this figure beside the tens of the root, and I also write it at the

right hand of the number which is below; then, looking upon 44 as the divisor, and 4 as the quotient, I perform the multiplication, and operate as in division; the subtraction being made, there remains 44.

Beside this remainder I write 54, which is the next period; then reasoning as above, I say, that 4454 contains only *the double product of the tens by the units, and the square of the units*; but the double product of the tens by the units, is a number of tens which has one place to its right; if then I separate one figure to the right of this number, the part 445 which remains to the left, ought to contain the double product of the tens by the units; I therefore double the tens which are now 24, which gives me 48, which I write below the root, after having barred the number 44 which served me before.

I divide 445 by 48, and I find for the quotient 9, which I write after the tens of the root, and beside the double of the tens by the units; I proceed as above, and have 53 for remainder; I bring down the last period, I separate the first figure to the right, I double the figures of the roots, to obtain a new divisor, which gives me 1 in the quotient, and 323 for remainder; so that the root of the greatest square contained in the number proposed, is 2491.

For the proof; the square of the root, joined to the remainder, must be equal to the given number.

The number 2491 is the square root of the number proposed within a unit, but if we wished to have it more exactly within a tenth, a hundredth, or &c. there would be no difficulty; we might continue the operation, after having added to the number twice as many cyphers as we wish to have decimals in the root; this manner of proceeding is founded upon

this principle, that the square of a number ought necessarily to contain twice as many decimals as there are in the root.

THE SQUARE OF FRACTIONS, AND

THE EXTRACTION OF THEIR ROOTS.

To obtain the square of a fraction, we must raise separately its numerator and denominator to their squares, since to square a fraction is to multiply it by itself. We see, then, that to obtain the square root of a fraction, we must take separately the square root of its two terms, and that might always be thus executed without embarrassment, if the fraction proposed were a perfect square.

But, when that is not the case, or rather when the denominator is not a perfect square, we must always render it such, by multiplying the two terms of the fraction by its denominator: then the root is always just within a fractional unit.

But the most simple method, when we are at liberty to employ it, is to express the value of the fraction in decimals, and to extend the division in such a manner, as to have twice as many decimals in the quotient, as we wish to have in the root.

If we have to extract the root of a square, which has an odd number of decimals, such as 45,7, we must supply the deficiency by putting a cypher to the right of the number, which does not change its value, and renders the number of decimals even, which is absolutely necessary, the number proposed being considered as a square.

EXTRACTION OF THE CUBE ROOT.

It is also on the knowledge we have of the composition of a *cube*, that is founded the method of the extraction of the *cube root* of any number whatever. But we must first observe, that every cube, which is not expressed by more than three figures, can have but one figure for its root; for 1000, which is the smallest number of 4 figures, has for its cube root 10, which is the smallest number of two figures; thus every number below 1000, will have but one figure for its root; then to find the cube root of a number of three figures and below, there is no necessity of method, we must have recourse to a table which contains the cube of all the simple numbers; the question then is, to find the cube root of a number composed of more than three figures.

This number ought necessarily to contain two figures in its root; let us see, then, how a cube is formed, which contains tens and units in its root; let us suppose this root to be 24; I form at first its square, which gives for the value of

	Square of the units.	*Double product of the tens by the units.*	*Square of the tens.*
the square,	16 - - -	160 - - -	400

We must, to have the cube, multiply each of the parts of the square, by the tens and the units of the root; beginning with multiplying all the parts of the square, by the units, and then by the tens; let us also set down separately all the partial products; we shall have six products.

Cube of the units.	Twice the tens by the square of the units.	Square of the tens by the units.	The tens by the square of the units.	Twice the square of the tens by the units.	Cube of the tens.
1st - 64 -	2nd 640 -	3d 1600 -	4th 320 -	5th 3200 - -	6th 8000

I now resume all these products, beginning by the right, and put together the 5th and the 3d, as

also the 4th and the 2nd; thus I see, that a cube is composed of 4 parts; viz. *of the cube of the tens*; *of 3 times the square of the tens by the units*; *of 3 times the tens by the square of the units*; and, lastly, *of the cube of the units*.]

With this knowledge, we can find the cube root of any number whatever.

Suppose we have the number - - - - - - I remark, first; that this number, being composed of more than three figures, contains tens and units in its root, and is, therefore, composed of the cube of the tens, of 3 times the square of the tens by the units, of &c. but the cube of the tens is a number of thousands, which has three places to its right; therefore if we separate three figures to the right of the number proposed, the part on the left will contain the cube of the tens.

94 | 897 | 584	456
64	——
———	48
308,97	6075
91125	
———	
37725,84	
94818816	
———	
78768	

But this number, being itself composed of more than three figures, contains also tens and units in its root, and is, consequently, composed of the cube of the tens, of three times the square of the tens by the units, of &c. therefore, if we cut off three figures towards the right of this number, the part on the left will contain the cube of the tens; and, as this part contains less than 4 figures, I can know the root by means of the table of the cubes of the single figures.

I see that the root of the greatest cube, comprised in this number, is 4; I write this figure as we see, then I subtract its cube from 94, and to the remainder 30 I bring down the next period.

Now I consider, that the number 30897, contains only three parts of the cube, since from the number 94897, we have subtracted the cube of the tens; these three parts, are therefore *three times the square of the tens by the units, more three times the tens by the square of the units, and the cube of the units.*

But three times the square of the tens by the units, is a number of hundreds, which has two places to its right; if, therefore, we separate two figures to the right of the number of which we are speaking, the part on the left will contain three times the product of the square of the tens by the units; and, if we divide this number by three times the square of the tens, we shall have the units in the quotient, 48 is three times the square of the tens; we must then make use of it to divide 308.

The most expeditious method of trying the figure of the quotient, is to cube the root, supposed to be known, and to subtract this cube from the number, the root of which it represents; we shall find here, that 5 is the proper quotient; having then subtracted the cube of 45, from the number 94897, we must bring down to the remainder the next period, and continuing to operate, as we have just done, we shall find, that the figure of the units is 6, so that 456 is the cube root of the number proposed, with a remainder of 78768.

If we were desirous of having the root of this number more exactly, within a tenth, or a hundredth &c. we must continue the operation, by placing to the

right of this number, three times as many cyphers as we wish to have decimals in the root, since the cube of a number, ought to contain three times as many decimals as its root.

As to the extraction of the cube roots of fractions, we are to be conducted by the same method of reasoning, and according to the same principles, which we have laid down for obtaining their square root.

If we have to extract the cube root of a number which contains decimals, but which cannot be separated into periods of three decimals, we must supply the deficiency, by putting cyphers to the right hand of the number.

ELEMENTS

OF

ALGEBRA.

K

ALGEBRA.

ALGEBRA treats of magnitude in general. It is a kind of universal Arithmetic. The characters therein employed are the letters of the alphabet, which, having no determined value, are thereby very proper to represent all kinds of magnitude or quantities. We perform upon algebraical quantities, the same operations as on numerical ones; we add, subtract, multiply, and divide them, but as these operations *can only be indicated, in quantities purely algebraical,* we have had recourse to certain signs, *the use of which is an invention which particularly characterizes Algebra.*

The sign of Addition is $+$, and is pronounced *more*.

The sign of Subtraction is $-$, and is pronounced *less*.

The sign of Multiplication is $\times$, and signifies *multiplied by*; we often put only a dot between the factors of the multiplication, and we sometimes also write the letters beside each other, without separating them by any sign; so that $a \times b$, or $a \cdot b$, or ab, express the same thing.

The sign of Division, is, as in arithmetic, a horizontal bar between the dividend and divisor; we some-

times also put two dots between the two quantities; so that $\frac{a}{b}$, or $a : b$ express the same thing.

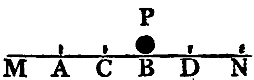
We call a *monomial*, every quantity, the constituent parts of which are not separated by the sign $+$, or the sign $-$; and we call a *polynomial* every quantity composed of several *monomials*, separated by the sign $+$ or the sign $-$. A *binomial* is a *polynomial*, composed of two *monomials*, or of two terms ;* a *trinomial* of three, &c.

Algebra, in its generality, comprehends indifferently *positive* and *negative* quantities. It is easy to conceive what is meant by a *positive* and a *negative* quantity.

Let us consider the condition of a man who has property and debts ; it is evident, that if I count his property as a *positive* quantity, I ought on the contrary to look upon his debts as *negative* quantities, since the latter tend to diminish the former.

If then I call a his property, and b his debt, his situation will be expressed by $a-b$.

Let us also consider a body P in motion on the line M N, and suppose we wish to compute its distance, with relation to the



point A; if the body move towards N, and be arrived at the point D, its distance from the point A will be

* We also give the name of *term*, to a quantity, such as $\frac{a+b}{d}$, though it is not a monomial: In general, every quantity, such as $\frac{a+b-c+\&c.}{m+n+\&c.}$, joined by the sign of division, is called a term in the same manner as this, $(a+b+\&c.)\times d$.

expressed by AB+BD ; but if, on the contrary, the body be in motion towards M, and be arrived at the point C, its distance from the point A will then be AB—CB ; we see, therefore, that if in the first supposition the course BD performed by the body P, be taken as a positive quantity, in the second supposition the course BC, performed by the same body, must be taken as a negative one.

This distinction being once established, it was essential to know what sign ought to effect the result of the combinations, which we could make of positive quantities with negative ones ; this is what we are going to explain, by giving the rules of Addition, Subtraction, Multiplication and Division.

ADDITION.

Addition of algebraical quantities, consists in writing the quantities one after another, and in preserving their signs such as they are. Thus to add $+b$ and $+a$, we must write $a+b$, (*every quantity which is not preceded by any sign, is always supposed to have the sign* +) ; and to add $-b$ & a, we must write $a-b$. The reason of this rule, results from what we have said, that b preceded by the sign —, being considered as a debt, it is evident, that to add a debt to an estate, is to diminish the estate, so that a being the estate and b the debt, to add them together we must write $a-b$. If we had to add a and a we would then write $a+a$, but we abridge this expression by putting $2a$; and in like manner, if we had $2a+5a$, we would write $7a$. The figure written before a letter or a quantity, is called the *coefficient* of that letter or of that quantity.

We shall remark, that the unit is always supposed to be the *coefficient* of every quantity which has none, for a or $1\times a$, is the same thing.

SUBTRACTION.

Subtraction of algebraical quantities, is performed by changing all the signs of the quantity to be subtracted; that is to say, that $+b$ to be subtracted from a equal $a-b$, and that $-b$ to be subtracted from a equal $a+b$; for, the result of the subtraction, joined to the quantity subtracted, must be equal to the other quantity; whence it follows, that if in the 1st example, to the difference $a-b$ we add the quantity subtracted $+b$, we must have a for the result; in fact $a-b+b=a$; in like manner if, in the second example, to the difference $a+b$ we add $-b$ the result must be a, what happens in effect, for $a+b-b=a$, therefore, &c.

We plainly see, that if we had $5a$, from which we wished to subtract $2a$, instead of writing $5a-2a$, we would simply write $3a$.

Example. *Let it be proposed to subtract* $4a-b+c$ *from* $5a-3b-c+d$: we must then change the signs of the quantity to be subtracted, which is here the first that presents itself, and write it after the second; which gives $5a-3b-c+d-4a+b-c$; *this is the result of the subtraction*; but we can simplify this result, by reducing the like quantities; and then we have $a-2b-2c+d$. We must never neglect making these kinds of reductions after addition or subtraction.

MULTIPLICATION.

There are three rules to be observed in multiplication, that of the Signs, that of the Coefficients, and that of the letters.

The rule of multiplication for the Letters, is to write them one after another; but when a same letter is to

be taken several times as a factor in a monomial, it has been agreed upon to write this letter but once, with a little figure raised above it on the right hand, which expresses the number of times that this letter ought to be taken as factor; thus for example $a.\ a.\ a$, I write a^3. this little figure placed above, is called *exponent*; we must take great care not to confound it with the coefficient.

If we had $a^2 \times a^3$, which is the same thing as $aa \times aaa$, we would then write a^5; whence we see, that multiplication of like letters or quantities, is performed by adding together their exponents; thus $a^4\ bc^3d \times ab^2c^2 = a^5b^3c^5d$, considering that unity is always supposed to be the exponent of every quantity which has none; for b is evidently the same thing as b^1.

What we have just said, only regards a monomial to be multiplied by a monomial; but with a little reflection, we shall see clearly that its application is easy, when a polynomial is to be multiplied by a monomial; and that we shall not be more embarrassed, to multiply a polynomial by a polynomial; for all the parts of the multiplicand, being to be multiplied by each part of the multiplier, we see plainly that to form all the partial products which compose the total product, we shall never have but a monomial, to be multiplied by a monomial:

The rule of multiplication for the Coefficients, consists in multiplying them by one another, according to the rules of arithmetic. Thus $7ab \times 9bd = 63ab^2d$.

The rule of multiplication for the Signs, requires a particular demonstration; we can form four different combinations with the signs $+$ and $-$.

We must prove - 1st. that $+ \times + = +$
2nd. that $- \times + = -$
3d. that $+ \times - = -$
4th. that $- \times - = +$

As to the first case, it stands in need of no demonstration; it is evident that $+a\times+b=+ab$.

To demonstrate the second case, let us suppose we have the binome $\overline{b-c}\times a$; if we perform the multiplication without paying attention to the signs, we shall have the two products ab and ac; but in multiplying b by a, it is evident, that I have multiplied a quantity too great by c, for it was not b that I had to multiply, but b diminished by c; then as many times as I have taken b, so many times have I taken c too much; but I have taken b,a times too much, therefore from the product ab, we must subtract c taken a times, or ac and then write $ab-ac$; thus $\overline{b-c}\times a=ab-ac$, and $-c\times+a=-ac$, therefore $-\times+=-$.

The third case falls under the second, because we can take indifferently either of the factors for multiplicand.

The fourth case is demonstrated thus. Let there be $\overline{a-b}\,.\,\overline{c-d}$; let us first multiply a by $\overline{c-d}$, according to what we have just demonstrated, we shall have $ac-ad$; but it was not a that I had to multiply by $\overline{c-d}$, but a diminished by b; thus, as many times as I have taken a, so many times have I taken b too much; but I have taken a, $\overline{c-d}$ times, I must then subtract from it b taken $\overline{c-d}$ times, or $bc-bd$, and *as to subtract, we must change the signs*, I shall then have $\overline{a-b}\,.\,\overline{c-d}=ac-ad-bc+bd$; we see that the product of $-b$ by $-d=+bd$; therefore $-\times-=+$.

EXAMPLES.

First.		Second.
$a^3+3a^2b+3ab^2+b^3$	*multipliers,* - - -	$5a^3-4a^2b+5ab^2-3b^3$
$a^3-3a^2b+3ab^2-b^3$	*multiplicands,* - -	$4a^2-5ab+2b^2$
$a^6+3a^5b+3a^4b^2+a^3b^3$	*partial products.*	$20a^5-16a^4b+20a^3b^2-12a^2b^3$
$-3a^5b-9a^4b^2-9a^3b^3-3a^2b^4$		$-25a^4b+20a^3b^2-25a^2b^3+15ab^4$
$+3a^4b^2+9a^3b^3+9a^2b^4+3ab^5$		$+10a^3b^2-8a^2b^3+10ab^4-6b^5$
$-a^3b^3-3a^2b^4-3ab^5-b^6$		
$a^6-3a^4b^2+3a^2b^4-b^6$	*total products,*	$20a^5-41a^4b+50a^3b^2-45a^2b^3+25ab^4-6b^5$

We have multiplied all the terms of the multiplicand by each of the terms of the multiplier, beginning at the left hand; and we have formed as many lines of partial products, as there are terms in the multiplier. We have written all the lines underneath one another; then, to form the total product, which is found written below, we have made the reduction of the similar terms; to facilitate this reduction, and to avoid mistakes, we bar the terms of the partial products, as we write them in the total product, and as they destroy one another.

DIVISION.

There are four rules to be observed in division, that of the Signs, that of the Coefficients, that of the Letters, and that of the Exponents.

The rule of Signs is easily found by the result of the signs of multiplication; in fact, since the divisor, multiplied by the quotient, must produce the dividend, it is evident, - - - - -

1st. that $\frac{+}{+} = +$

2nd. that $\frac{-}{+} = -$

3d. that $\frac{+}{-} = -$

4th. that $\frac{-}{-} = +$

Let us give out the reasoning for one case only, and let us take the second. I say then, that — divided by + gives less, for if I multiply + the sign of the divisor by — the sign of the quotient, I shall have —, which is the sign of the dividend, therefore *less* ought to be the sign of the quotient.

As to the rule of Coefficients, we must divide them by each other according to the principles of arithmetic; then write the quotient instead of the dividend, and suppress the divisor, or write 1 in its place; but we must observe, that this division is performed only when it can be made without remainder; otherwise the quotient, being composed of two parts, the quantity would then be presented under a complicated form; in this case we only indicate the division, and write the numbers such as they are before the letters.

Thus $\frac{6a}{2b} = \frac{3a}{b}$ ...... $\frac{2a}{6b} = \frac{a}{3b}$ ...... $\frac{4}{8a} = \frac{1}{2a}$; but $\frac{5a}{4b}$ will not admit any change.

For the rule of Letters, we must efface the similar letters in the dividend and divisor, which has no need of demonstration, since in multiplication we are satisfied with writing them one after another. Thus $\frac{abc}{bcd} = \frac{a}{d}$

For the rule of Exponents; when there are similar letters or quantities in the dividend and divisor, with different exponents, we must subtract the less exponent from the greater, then write this difference instead of the greater, and efface the letter, or the quantity, which is found to have the less exponent.

Thus $\frac{9a^3bc^5}{3ab^3c^2} = \frac{3a^2c^3}{b^2}$; this proceeding is a consequence of the rule established in multiplication for what relates to exponents. What we have just said, teaches us to divide a monomial by a monomial, which would be equally applicable to the case in which we should have a polynomial to be divided by a monomial; since the operation would then be reduced, to divide separately each of the terms of the polynomial by the monomial; but if we had a polynomial to be divided by a polynomial, the following method must be pursued.

We begin by *arranging* the dividend and divisor with respect to a same letter. The arranging a polynome consists only in writing the terms of the polynomial in such an order, that the first may contain the letter by which we arrange, raised to its highest exponent; the second may contain the same letter rai-

sed to an exponent next less, &c. That being performed in both numbers, we divide the first term of the dividend by the first term of the divisor, we write the quotient under the divisor; then we multiply all the divisor by the quotient, to subtract it from the dividend; we proceed in this according to the principles of division of numbers.

Suppose we have $6a^2b^2+4ab^3+b^4+a^4+4a^3b$ to be divided by $2ab+b^2+a^2$.

I arrange the dividend and divisor with respect to the same letter a, and I then write;

$$
\begin{array}{l l|l}
 & a^4+4a^3b+6a^2b^2+4ab^3+b^4 & a^2+2ab+b^2 \\
 & -a^4-2a^3b-a^2b^2 & \overline{a^2+2ab+b^2} \\
\hline
\text{1st } rem. & 2a^3b+5a^2b^2+4ab^3+b^4 & \\
 & -2a^3b-4a^2b^2-2ab^3 & \\
\hline
\text{2nd } rem. & \quad a^2b^2+2ab^3+b^4 & \\
 & -a^2b^2-2ab^3-b^4 & \\
\hline
\text{3d } rem. & \quad 0 \quad 0 \quad 0 &
\end{array}
$$

We divided a^4 by a^2, then we multiplied all the divisor by the quotient a^2; we wrote all the products under the dividend, with the attention of changing their signs; we obtained a first remainder; we divided the first term of this remainder by the first term of the divisor, which gave $2ab$ in the quotient; we then multiplied the divisor by this new partial quotient; then, &c.

Another Example, in which the two members of the division are arranged.

EXAMPLE.

$$
\begin{array}{rl}
20a^5-41a^4b+50a^3b^2-45a^2b^3+25ab^4-6b^5 & \left| \; 5a^3-4a^2b+5ab^2-3b^3 \right. \\
-20a^5+16a^4b-20a^3b^2+12a^2b^3 \qquad\qquad\qquad & \left| \; \overline{4a^2-5ab+2b^2} \right. \\
\hline
\text{1st } rem. \; -25a^4b+30a^3b^2-33a^2b^3+25ab^4-6b^5 & \\
+25a^4b-20a^3b^2+25a^2b^3-15ab^4 \qquad & \\
\hline
\text{2nd } rem. \; \text{- - - -} +10a^3b^2-8a^2b^3+10ab^4-6b^5 & \\
-10a^3b^2+8a^2b^3-10ab^4+6b^5 & \\
\hline
0 \qquad\qquad\qquad &
\end{array}
$$

It often happens, that the division cannot be performed exactly; then we must, as in arithmetic, write the remainder under a fractional form by the quotient.

FRACTIONS.

The same rules are to be followed in the calculation of algebraical fractions, as in those of numbers.

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But to reduce fractions to the same denominator, there is a case, which it is proper to examine relatively to the signs. Let it be proposed to reduce the two terms of the quantity $\frac{a}{a-d}-\frac{a-b}{a+b}$ to the same denominator; by operating according to the principles of arithmetic, we shall have $\frac{a\times\overline{a+b}-\overline{a-b}\times\overline{a-d}}{(a-d).(a+b)}$; effectuating the calculation, the quantity will become $\frac{a^2+ab-a^2+ab+ad-bd}{a^2-ad+ab-bd}$, which is equal, after making the reduction, to $\frac{2ab+ad-bd}{a^2-ad+ab-bd}$.

We must remark, relatively to the signs, that, having made the multiplication of $\overline{a-b}$ by $\overline{a-d}$, we have changed all the signs of the product, because the sign $-$, which is before the second term of the quantity proposed, indicates that this second term is to be subtracted from the first, (the algebraical subtraction consisting in changing the signs of the quantity to be subtracted.) In effectuating the multiplication of $\overline{a-b}$ by $\overline{a-d}$, we said $+a$ multiplied by $+a$, gives $+a^2$; but as it must be subtracted, we have written $-a^2$; then $-b$ multiplied by $+a$, gives $-ab$, but as it is to be subtracted, we have written $+ab$, &c.

In such cases, beginners are apt to make mistakes in fixing the signs: Let them consider, that every compound term, preceded by a sign, is governed by that sign. Suppose, for example, the quantity $a-(b+c-d)$ if we take away the parenthesis, we must write $a-b-c+d$, for as the whole quantity, included by the parenthesis, is to be subtracted, we must change the signs of it.

Nevertheless we will observe that if $\frac{b}{c-d}$ is to be subtracted from a, we must write $a-\frac{b}{c-d}$, and not change the signs of the denominator; because every fraction being the expression of a quotient of a division, the sign of the quotient is not changed, if we change, at the same time, the signs, both of the dividend and the divisor, since $+$ divided by $+$, and $-$ divided by $-$, both give $+$ for the sign of the quotient.

EXPONENTS.

It is very necessary to be well acquainted with the calculations that may be performed with like quantities or like letters, by means of their exponents. Under this head, we shall explain all that belongs to the theory of exponents, and partly repeat what has been already said under the heads of multiplication and division. From this theory we learn,

1st. That like quantities, or like letters, are multiplied the ones by the others, by adding their exponents; thus for example, $a^3 \times a^2 = a^{3+2} = a^5$; for $a^3 \times a^2$ is in fact the same as $aaa \times aa$, or a taken five

times as factor, which it has been agreed on to express by a^5. According to the same rule, $a^3bc^2 \times a\,b^2c^4 = a^4b^3c^6$, ($b$ being the same thing as b^1.)

2ndly. That like letters are divided the one by the other, by subtracting their exponents; thus $\frac{a^5}{a^3} = a^{5-3} = a^2$; for $\frac{a^5}{a^3}$ is the same thing as $\frac{aaaaa}{aaa}$, which, by casting off, or expunging the letters, common to both dividend and divisor, is found equal to $aa = a^2$; by working in the same way, we shall find $\frac{a^5bc^4}{a^3b^3c^3} = \frac{a^2c}{b^2}$.

From this it follows;—1st. That a quantity raised to no higher power than zero equals unity; thus, for example, $a^0 = 1$; for $\frac{a}{a} = \frac{a^1}{a^1} = a^{1-1} = a^0$; but $\frac{a}{a} = 1$, therefore $a^0 = 1$.——2ndly. That every quantity, whose exponent is negative, is equal to unity divided by that quantity, with the same exponent, but positive. Thus, $a^{-2} = \frac{1}{a^2}$; for $\frac{a}{a^3} = a^{1-3} = a^{-2}$; but $\frac{a}{a^3} = \frac{1}{a^2}$, therefore $a^{-2} = \frac{1}{a^2}$.

From this we conclude, that a divisor may, at pleasure, be changed into a multiplicator, and a multiplicator into a divisor; for $\frac{a}{b} = ab^{-1}$; and $\frac{a^3}{b} = \frac{1}{ba^{-3}}$.

3dly. That, to raise any quantity as a^n to any power as m, the exponent of that quantity must be multiplied by m, which will give a^{mn}; for, in order to raise a^n to the power m, a^n must be taken m times as factor, thus we have $a^n \times a^n \times a^n \times$ &c. which is equal to $a^{n+n+n+\&c.}$, since the multiplication of like quantities is performed by the addition of their exponents; then we see, that the exponent of a is n, taken as often as the number of the factors; but this number of factors is m, therefore we must take n, m times, and then write a^{mn}.

In the same way, if a^2b^3c is to be squared, or raised to the second power, which is expressed thus, $(a^2b^3c)^2$, the exponent of each factor of this quantity must be multiplied by 2, which will give $a^4b^6c^2$, for $(a^2b^3c)^2 = a^2b^3c \times a^2b^3c = a^4b^6c^2$.

4thly. That to extract any root, such as m, of any quantitty such as a^n the exponent of this quantity must be divided by m, which will give $a^{\frac{n}{m}}$. This rule is a necessary consequence of the preceding one: for it is evident, that, in order to obtain any root of a quantity, we must pursue a method directly contrary to that which is followed to raise it to any power. Thus if we wished to extract the square root of a^2b^3c, since in squaring it, we multiplied the exponent of each of its factors by 2, it is evident, that in this case, we ought to divide these same exponents by 2, which will give $ab^{\frac{3}{2}}c^{\frac{1}{2}}$

RADICALS.

The sign $\sqrt{}$, *called by mathematicians a radical sign*, when placed over a quantity, designates some root of that quantity. In the fork of this radical is placed a

number which designates the particular root to be extracted. Thus $\sqrt[2]{a}$ expresses the square root of a; but it has been agreed, to write simply $\sqrt{a}$ when none but the square root is required.——$\sqrt[3]{a}$ expresses the cube root of a ——$\sqrt[4]{a}$ its biquadratic root, and so on with the roots that follow. We call *Radical quantities* those which include *Radicals*, and those *Rational*, which include none.

We work with *Radical quantities* in the same way as we do with *Exponential quantities*; because every radical quantity can be readily put into an exponential form; for we have just seen that any root, such as m, of any quantity, such as a^n, when expressed in the radical form $\sqrt[m]{a^n}$, is the same thing as the *exponential quantity* $a^{\frac{n}{m}}$.

If the radical quantity $\sqrt[3]{a^2b^5c^3d}$ was to be put into an exponential form, it would be written thus, $a^{\frac{2}{3}}b^{\frac{5}{3}}\cdot c\cdot d^{\frac{1}{3}}$; the *exponent*, of each factor of the quantity under the radical, being divided by the *index* of the root.

As we can at pleasure, put a *radical quantity* under an *exponential form*, so we can with equal facility, put an *exponential quantity* under a *radical form*, if for example, the whole quantity $a^2\sqrt[3]{b}$ was to be put under the *radical*, it would be written thus, $\sqrt[3]{a^6b}$, because we must multiply the exponent of the quantity to be put under a radical, by the index of this radical, which is easily understood.

From this, it follows, that we may sometimes simplify the quantity under the radical. If we had for

example, $\sqrt{16a^3}$; we might put this quantity under this form, $4a\sqrt{a}$, by taking the square root of 16, and, as $a^3=a^2a$, by taking the square root of a^2, and leaving a under the radical. In case the coefficient under the radical be not a perfect power of the index of the radical, we could yet sometimes render the quantity under the radical more simple, by dividing this coefficient into two factors, one of which must be a perfect power. Example. $\sqrt{20a^5}=\sqrt{4\times 5a^4 a}=2a^2\sqrt{5.a}$ ——other example, $\sqrt[3]{24a^4}=\sqrt[3]{8\times 3a^3a}=2a\sqrt[3]{3a}$.

If we had the quantity $\sqrt{a^2+b^2}$, we must take care not to extract separately, the square root of a^2 and b^2, because, as it will soon be seen, $a+b$ raised to the second power, is not equal to a^2+b^2.

All the calculations to be made upon radicals, will be then always easily performed, since, in putting, in reality, or by thought, the radical quantities into an exponential form, we must follow the same rules that have been laid down for rational quantities.

We will say no more under this head. Practice is necessary in this kind of calculation, to perceive, at the first glance, the different tranpositions and combinations of which it is capable.

INVOLUTION.

Involution consists in raising a quantity to any proposed power.

From what has been just said concerning exponents, it is evident, that, in order to raise a monomial to any given power, nothing more is necessary than

to multiply the exponent of each factor of the monomial, by the index of the power. But the method to be pursued, to raise a polynomial to its different powers, is not so simple; for, if in the first place, we wish to have the square, we must multiply the polynomial by itself; then the cube is obtained, by multiplying the square by the root; we have biquadrate by multiplying also the cube by the root, and so on; so that, to raise a polynomial to any power, all its anterior powers must be found first; an operation which is often long and tedious.

But there is a rule, invented by Newton, very convenient, and easy to be remembered, for raising a binomial to any power; and it can be readily applied to any polynomial. This rule is known by *the binomial rule, or the Orem of Newton.* The principles on which it is founded, will now be explained.

Let it be required to multiply several binomials into one another, and let us take only four; this number will be sufficient for our purpose. Be then

$\overline{x+a} \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}$; upon performing this multiplication, the total product will come out thus, $x^4+(a+b+c+d)x^3+(ab+ac+ad+bc+bd+cd)x^2+(abc+abd+acd+bcd)x^1+abcdx^0$, which is commonly put under the following more convenient form.

$$\begin{array}{llll} x^4+a\,.\,x^3 & +ab\,.\,x^2 & +abc\,.\,x^1 & +abcdx^0. \\ \quad +b\,. & +ac\,. & +abd\,. & \\ \quad +c\,. & +ad\,. & +acd\,. & \\ \quad +d\,. & +bc\,. & +bcd\,. & \\ & +bd\,. & & \\ & +cd\,. & & \end{array}$$

In the first place, it will be remarked, that the first term x of each binomial, is found in every term of

the total product, and that, its exponent decreases continually by unity, beginning from the first term, whose exponent 4, expresses the number of binomials employed in forming this product.

In the next place, that with respect to the coefficients of x, in the several terms, it will be seen, that the coefficient of the first term x, is unity; that of the second, is the sum of letters a, b, c, d; that of the third, is the sum of the products arising from combining those letters, two by two; that of the fourth, is the sum arising from combining them three by three; and that of the fifth, is the product of all the letters; but if there had been a greater number of factors, the coefficient of the fifth term would have been the sum of the products of all the letters taken four by four; and in the same proportion with the other terms, but so that the last term always consists of the product of all the letters.

Let this result be generalised; let m be the number of binomials; and let us suppose, at the same time, that all the binomials are equal, and consequently the letters b, c, d, &c. equal to a; then the expression of the multiplication, will be $\overline{x+a} \times \overline{x+a} \times \overline{x+a} \times \overline{x+a}$ &c; that is to say, $x+a$ taken m times as factors, or $(x+a)^m$.

Thus, in order to raise any binomial to any power m, nothing more is necessary than to observe, after the supposition just made, what is the result of the total products we have developed.

The first term of this product, then, will be x^m; the coefficient of the second term, since all the letters

are equal, will be a taken as often as there are letters; but the number of letters is m, so that this coefficient will be $m\,a$, which, multiplied by x^{m-1} will form the whole expression of the second term.

The coefficient of the third term, since all the letters are equal, will be a^2 taken as often as the number of the products, that can be formed out of the given number of letters, by combining them two by two; but the number of these products, is evidently half the number of letters, which composed them; so that when the number of letters will be known, the number of products would be found by taking the half of them.

In order to find this number of the letters, it must be observed, that every letter should be combined with each one of the others; and as the number of the other letters is $m-1$, each letter must be then taken $m-1$ times; but as the total number of letters is m, the total number of letters which compose the products, will be $m-1$ taken m times, that is $m\,.\,\overline{m-1}$, which will give for the number of products $m\,.\,\frac{m-1}{2}$; then the whole expression of the third term will be

$$m\,.\,\frac{m-1}{2}\,.\,a^2\,x^{m-2}$$

The coefficient of the 4th term, since all the letters are equal, will be a^3 taken as often as we can form products out of a number m of letters, by combining them three by three; but the number of these products is the third of the number of letters which compose them; therefore when the number of letters are

known, to have the number of products the third of them must be taken.

In order to find this number of the letters, it must be observed that every letter must be combined with each of the products that can be formed by taking the other letters two by two, but the number of the other letters being $m-1$, we must first find the number of the products that can be formed out, by combining $m-1$ of letters two by two.

But we have just seen, that, when the number of letters was m, the number of the products was $m.\frac{m-1}{2}$; therefore when the number of letters is $m-1$, the number of the products will be $\frac{\overline{m-1}.\overline{m-2}}{2}$ which is the number of times that each letter should be taken; and as the number of letters is m, the total number of letters which compose the products will be $\frac{\overline{m-1}.\overline{m-2}}{2}$ taken m times, that is $m.\frac{\overline{m-1}.\overline{m-2}}{2}$; then the number of the products will come out $m.\frac{m-1}{2}.\frac{m-2}{3}$ and the whole expression of the 4th term will be $m.\frac{m-1}{2}.\frac{m-2}{3}a^3x^{m-3}$

By reasoning in the same way, we shall find that the expression of the fifth term will be $m.\frac{m-1}{2}.\frac{m-2}{3}.\frac{m-3}{4}a^4x^{m-4}$

Thus then, the Binomial $x+a$ raised to the power m, or $(x+a)^m = x^m + max^{m-1} + m \cdot \frac{m-1}{2} a^2 x^{m-2}$ $+ m \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} a^3 x^{m-3} + m \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} \cdot$ $\frac{m-3}{4} a^4 x^{m-4} +$ &c.

This process will be terminated, when, in applying it, having put in place of m its value, one of the coefficients of x becômes equal to cypher.

If, for example, we suppose $m=3$; then we see that the fifth term of the process will be cypher, since one of the factors of this term, $\frac{m-3}{4}=0$, and thus in the present example the process will consist but of four terms.

If the second term of the binomial was negative, had we $(x-a)^m$; the only difference in the process will be, that the sign $-$ must be placed before every term in which a stands with an odd exponent.

One more observation remains, which will render the raising of a binomial to any required power, extremely easy.

The coefficient of a term, of the process without comprehending in it the second term of the binomial,

which is found in every term of the process, with an exponent continually increasing by unity, beginning from the first term where its exponent may be supposed equal to cypher; *the coefficient of a term*, I say, is equal to the coefficient of the term immediately preceding, multiplied by the exponent of x in this term, and divided by the number of the terms which stand before the term we are writing.

For example, if the fifth power of $a+b$ is required; we have $(a+b)^5 = a^5 + 5ba^4 + 5 \cdot \frac{4}{2} b^2 a^3 + 5 \cdot \frac{4}{2} \cdot \frac{3}{3} b^3 a^2 + 5 \cdot \frac{4}{2} \cdot \frac{3}{3} \cdot \frac{2}{4} b^4 a^1 + 5 \cdot \frac{4}{2} \cdot \frac{3}{3} \cdot \frac{2}{4} \cdot \frac{1}{5} b^5 a^0$; the process ends here, the following term being cypher: the whole operation, with all its steps, has been laid down, that the application of the rule might be the better understood; but in practice, in proportion as we advance, we must reduce the coefficients of each term, as it comes. The calculation, being effectuated here, gives $(a+b)^5 = a^5 + 5ba^4 + 10b^2a^3 + 10b^3a^2 + 5b^4a + b^5$.

Be it proposed to square $\overline{x \pm a}$, we shall have $(x \pm a)^2 = x^2 \pm 2ax + a^2$; this simple example is given, only to afford an occasion to remark, that the square of a binomial consists *of the square of the* 1st *term of the root, more or less twice the second into the first, and of the square of the second.*

If it should be required to raise a polynomial to any power, as $(x + a + b + c + \&c.)^m$; we must, in the first place, take $a + b + c + \&c. = A$, and then we shall have $(x + A)^m = x^m + mAx^{m-1} + m \cdot \frac{m-1}{2} A^2 x^{m-2} + m \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} A^3 x^{m-3} + \&c.$ Now, putting in the place of the different powers of A, their

values, and making use of the same means that have been just employed, we shall have the total product of the polynomial raised to any power m.

EVOLUTION.

Evolution consists in extracting the root of any given quantity.

From the theory of exponents, it is evident, that there is no difficulty in extracting the root of a monomial; for, nothing more is necessary, than to divide the exponent of each one of its factors, by the index of the root to be extracted.

But the extracting of the root of a polynomial, requires particular rules, founded on a knowledge of the composition of the several powers of a binomial. Nevertheless, this operation is by no means difficult. The given quantity must, in the first place, be arranged; then we must take the root of the first term, and afterwards proceed according to the same rules that have been laid down for extracting the Square and Cube Roots in arithmetic.

In the mean while, it may be remarked, that, when the quantity to be worked on, is a perfect power of a binomial, the two terms of the root may be readily obtained, by extracting the desired root from the first and last term of the quantity, which is supposed to be properly ordered. This rule is proved in raising a binomial to its different powers.

Let the square root of $x^2 \pm 2ax + a^2$ be required, which we have seen above to be an exact power of

$x \pm a$; if the square root of x^2 is taken, we shall have x first term of the root, and, upon taking the square root of a^2, we have $\pm a$, the second term of the root.

As to the double sign before the second term of the root, it may be remarked, that, when the second term of the square is negative, the second term of the root will also be negative.

What has been just said of the extraction of the square root of a binomial, is applied immediately to solving equations of the second degree, and therefore deserves great attention.

The general rule, just given, can be followed, when an exact root is required; but when the quantity is such, that an exact root cannot be obtained, but only something near it, we must have recourse to the binomial rule of Newton, whose method for raising a quantity to an integral exponent, may be applied in the same way to fractionary exponents, so that $(x+a)^{\frac{m}{n}} = x^{\frac{m}{n}} + \frac{m}{n} a x^{\frac{m}{n}-1} + \frac{m}{n} \cdot \frac{\frac{m}{n}-1}{2} a^2 x^{\frac{m}{n}-2}$

$$+ \frac{m}{n} \cdot \frac{\frac{m}{n}-1}{2} \cdot \frac{\frac{m}{n}-2}{3} a^3 x^{\frac{m}{n}-3} + \&c.$$

Let $m = 1$, and $n = 2$; upon performing the operation, we shall have $(x+a^{\frac{1}{2}} = x^{\frac{1}{2}} + \frac{a}{2x^{\frac{1}{2}}} - \frac{a^2}{8x^{\frac{3}{2}}}$

$$+ \frac{a^3}{16x^{\frac{5}{2}}} - \frac{5a^4}{128x^{\frac{7}{2}}} + \&c.$$

This series would extend *ad infinitum*. In order to obtain a proximate root of $x+a$, we should simply take the first terms of the series, with this caution, that this means is never to be used, but when the terms of the series diminish continually; and it is evident, that the proximate value will be so much the more exact as the series decreases more rapidly, since then the terms that will be thrown away, will be trifling, and may be considered as nothing. It is very perceptible, that, if the terms of a series increased continually, it would not be possible to gain any thing near the value of that series.

Those series which continually decrease, are called *converging* ones; and those that continually increase, are called *diverging* ones.

The Binomial Rule can be readily applied to numbers, to extract any of their roots; in order to do it, the number should be divided into two parts, the first of which should be always greater than the second, and *an exact power of the required root*; which can always be done by making the second part positive or negative; then treating the two parts of this number, as the two terms of a binomial, the required root will be found.

We shall content ourselves with pointing out this operation, which is rather an object of curiosity than useful for the purpose of this work; as, in order to extract the square, or cube root of a number, the method taught in Arithmetic is more expeditious, and even, if the number is not an exact square, or cube, it is even better for obtaining a proximate root to make use of the decimal parts.

EQUATIONS.

An Equation is the equality of two or more quantities, separated by the sign $=$ *which signifies equal to.* Equations are of great use in solving *problems.*

A Problem is a question, the expression of which consists of *known* and *unknown* quantities, the latter of which are to be determined from the former.

Analysis is the art of solving problems; in order to effect it, the known must be combined with the unknown quantities, so as to form equations.

An *Equation* is solved, when the *unknown* quantity is found by itself in one of its *members*,* freed from all coefficients and exponents.

Equations are of different degrees. The highest exponent of the *unknown quantity*, determines the degree of the equation.——The *unknown quantities* are usually represented by the latter letters of the alphabet, u, t, x, y, z. Such an equation, as $x\,y = a$, is also of the second degree;——$x\,y\,z = b$ is of the third degree, &c.

That a problem may be soluble, the conditions of the problem must be such, that as many equations can be formed, as there are unknown quantities;

* The members of an equation are the two parts of it on the right and left of the sign of equality. The part on the right is called the first member; the other, the second member.

mark, that if the quantity b is added to each member of the equation, it will disappear from the first member; for then the equation, will be $x + a - b + b = d + b$, which is reduced to $x + a = d + b$.

Again, I remark that by subtracting a from each member of the equation, this quantity will disappear from the first member; for then the equation will be, $x + a - a = d + b - a$, and $x = d + b - a$. Here, then, is the solution of the equation; and if determined values were given to a, b, d, we should have a determined value for x.

If we reflect on the operations just performed, we may draw from them general rules. In order to disengage x, we see that addition and subtraction have been successively made use of; the principle of preserving the equality of the two quantities being constantly observed, *by adding to the one what we added to the other, and subtracting from the one what we subtracted from the other.*

But we see also, by the final equation $x = d + b - a$, that the result of those two operations is reduced to have a and b in the second member of the equation, with signs directly contrary to those they had in the 1st member; as the reasoning, by which we obtained this result, is not confined particularly to a and b, we conclude generally, *that in order to clear a member of an equation of any term, that term must be placed in the other member with a sign contrary to the one it had.* This rule affords also *a simple way of changing a term from one member to the other,* which is called *transposition.*

Let $3x - a = b + 2$ *be a given equation.*

In order to resolve this equation, all the terms in which x is found, must be transposed into the first member, and all the known quantities into the other; thus we shall have $3x - x = b + a$, and $2x = b + a$; finally $x = \frac{b+a}{2}$.

Let the given equation be $px - a = b + x$.

We shall have $px - x = b + a$, and $(p-1)x = b + a$, and finally $x = \frac{b+a}{p-1}$. Great attention should be paid to this manner of clearing the unknown quantity, when it is found in several terms of the equation, with different coefficients. *All the different coefficients of the unknown quantity should be united under the same parenthesis*, as was just now done; *then this assemblage should be considered as one coefficient.* This process is by no means difficult to understand, for upon performing the multiplication as it is expressed, it is evident, we shall have the same terms we had before this change of form.

With regard to the sign of each of the coefficients, comprised between the two same parentheses to form one coefficient, we must remark, *that each of these coefficients must preserve the sign it had before, if the sign + be before the parenthesis*, and on the contrary *if it be the sign —, the sign of each coefficient must be changed.*

An example will shew the necessity of this change.

Let it be $a^2 - bx - cx = d$.

In order to assemble the coefficients of x we must write $a^2-(b+c)x=d$; for, upon performing the indicated multiplication, and observing the rule for signs demonstrated under the head of fractions, we shall have the equation under its original form ; *in such case beginners are apt to mistake the signs.*

I continue solving the equation, and I have $-(b+c)x=d-a^2$; from which I draw $-x=\frac{d-a^2}{b+c}$; then changing the signs in both members, I shall have $x=\frac{a^2-d}{b+c}$.

Take notice here, that the signs of the denominator have not been changed, the reason of which has been already given.

Take the equation $\frac{x}{a}-\frac{b}{c}=\frac{d}{b}-\frac{b-x}{c}$.

According to the method laid down, we must begin with clearing the unknown quantity of its denominators ; the most simple way of doing it, is, in the first place, *to reduce to a common denominator the terms of each member separately, and then do the same by both members.*

We shall have in the first place, $\frac{cx-ab}{ac}=\frac{cd-b^2+bx}{bc}$, and as c is common to both the denominators, noth-

ing more is necessary, than to multiply the first member by b, and the second by a, which gives $\frac{bcx-ab^2}{abc} = \frac{acd-ab^2+abx}{abc}$. Now, it is very evident, that we can take away the denominators; for here are two equal fractions with the same denominator, consequently the numerators must be equal; thus we shall have $bcx - ab^2 = acd - ab^2 + abx$; let it be remarked, that in the two members, there are two like terms with the same signs, *which may therefore be thrown away without injuring the equality*: Then the equation is reduced to $bcx = acd + abx$, and by transposition $bcx - abx = acd$, or $(bc - ab)x = acd$, from which, finally, $x = \frac{acd}{bc-ab}$.

Let us propose again to solve this equation, $\frac{a+x}{x-\frac{b}{c}} - d = \frac{b}{d}$.

Before proceeding to clear away the denominators, the first term of the equation requires some preparation; its denominator, which is composed of a whole quantity and a fraction, should be entirely reduced under the fractional form, and then it becomes $\frac{cx-b}{c}$; thus we have, therefore, $a + x$ to divide by a fraction; that is to say, according to a principle of arithmetic, this quantity must be multiplied by the denominator of the fraction, and the product must be divided by

the numerator; the proposed equation will then become $\frac{(a+x)c}{cx-b} - d = \frac{b}{d}$; *now the terms of the first member must be reduced to a common denominator*, and we shall have $\frac{ac+cx-cdx+bd}{cx-b} = \frac{b}{d}$, then both members of the equation must be so reduced, which gives, after having effaced the common denominator, $acd+cdx-cd^2x+bd^2 = bcx-b^2d$, and $cdx-cd^2x-bcx = -b^2d-acd-bd^2$; or $(cd-cd^2-bc)x = -b^2d-acd-bd^2$; finally, $x = \frac{-b^2d-acd-bd^2}{cd-cd^2-bc}$, a value which presents itself under a negative form, but which can be expressed under a positive one, by changing the signs of all the terms of the numerator and the denominator, and then we shall have $x = \frac{b^2d+acd+bd^2}{cd^2+bc-cd}$.

By becoming very familiar with the examples just given, a learner will not find it difficult to solve every kind of problem of the first degree with one unknown quantity, when the conditions of the problem shall have been expressed by an equation; but, to find this equation, we cannot give a method; a certain sagacity, as has been already said, and a great deal of practice, are necessary; nevertheless, the following rule can aid us.

We must always begin, by representing the unknown quantity by a letter of the alphabet, x for example;

then reasoning, as if this quantity was known, we find, by reflecting on the conditions of the problem, a combination between the known and unknown quantities, which necessarily furnishes an equation.

PROBLEM. *A labourer saved* 30 *cents from the price of his work each day, and, on the contrary, he spent* 70 *cents every day he did not work; at the end of* 90 *days he finds he has but a dollar of what he had saved. We would wish to know how many days he passed without working?*

We will call, then, x the number of the required days. Now, studying the question, I discover in it an equality; for, as the expence of this man has been taken from what he saved, and no more remains to him but a dollar, or a hundred cents; it is therefore evident, that his expence, more a hundred cents, equals the whole of what he had saved; nothing more is necessary then, than to express this equality.

Now this man expending 70 cents every day that he did not work, and this number of days being x, his whole expence will be expressed by $70x$; but the whole number of days being 90, $90-x$ will be evidently the number of days he worked, on each of which this man saved 30 cents; thus the whole he saved should be expressed by $30(90-x)$. Then we have from thence $70x+100=30(90-x)$; dividing all the terms by 10, performing the multiplication and transposition, the equation becomes $10x=260$ and $x=26$, *the number of days which the labourer has not worked.*

For the proof, it is necessary to substitute the value of the unknown quantity into the first equation, which

ought to come out identical if the work has been done right. Putting then 26 in place of x, we have $70 \times 26 + 100 = 30\ (90-26)$, an identical equatión, for each member becomes 1920.

We will observe, however, that the proof serves but to ascertain the exactness of the work; for, if the equation was false, and not conformable to the conditions of the problem, it is evident that the solution would be false, though the proof was just.

EQUATIONS OF THE FIRST DEGREE,

WITH SEVERAL UNKNOWN QUANTITIES.

If the problem contains several unknown quantities, it is necessary to form as many equations as there are unknown quantities, then to combine these different equations in such a manner, that there may result from them one équation, with but one unknown quantity; having found the value of this, it is evident that by substitution, the value of the rest may be easily had. The only difficulty, then, which presents itself, is, to obtain this single equation, which shall contain but one unknown quantity. The method of destroying the other unknown quantities is called *extermination*. There are three rules for the *extermination* of unknown quantities, but we shall mention but one, which is the most expeditious, and which can be employed in every case.

This rule consists in comparing the equations two and two, and in treating them in such a manner that the unknown quantity, which we wish to exterminate, may have the same coefficient in the two equations; then adding or subtracting the one from the other, this unknown quantity will disappear, and we have an equation, which contains one unknown quan-

tity less, consequently if the problem has in all but two unknown quantities, this equation will contain but one unknown quantity, and it will be solved according to the principles already given.

But if the problem contains three unknown quantities, and consequently three equations, it will be necessary to continue the operation, by comparing the first or second equation with the third, from whence will result a new equation, which will contain no more, than the same two unknown quantities, contained in the equation obtained just before it; from the combination of those two, a single equation will be obtained, which will contain but one unknown quantity, the value of which being found, shall be substituted into one of the equations, with two unknown quantities; then, having the value of these two, we will substitute them both into one of the three equations of the problem, and we shall have the value of the third unknown quantity. It is evident what method we must pursue, if there should be a greater number of equations. We will now render all those processes evident by general examples, then we will give some problems.

Let the two equations be

$$ax + by = d$$

and

$$mx + ny = p$$

SOLUTION.

$$amx + bmy = md$$

and

$$amx + any = ap$$

$$bmy - any = md - ap$$

$$(bm - an)y = md - ap$$

$$y = \frac{md - ap}{bm - an}$$

$$ax + b \cdot \frac{md - ap}{bm - an} = d$$

$$ax = d - b \cdot \frac{md - ap}{bm - an}$$

$$x = \frac{d - b \cdot \dfrac{md - ap}{bm - an}}{a}$$

$$x = \frac{bdm - adn - bdm + abp}{abm - a^2 n}$$

$$x = \frac{abp - adn}{abm - a^2 n}$$

$$x = \frac{bp - dn}{bm - an}$$

Explanation of the Operations.

Wishing to exterminate x, we multiplied all the terms of the first equation by m the coefficient of x in the second, and all the terms of the second by a the coefficient of x in the first. Then we subtracted the second from the first, by which x was destroyed, and there remains but y, whose value was found.

To have the value of x, we have substituted the value of y into the first given equations; it can be found equally well by substituting the value of y into the second.

We must here make use of this second equation for the proof; since by substituting into it the found values of x and y, it is evident that this equation ought to be identical, if the work had been done right.

If one of the equations contains but one unknown quantity, the calculation will be the same but more simple, and the same method is followed: for example, *Let the two equations* - - - $ax - by = d$
and $ny = p$

Explanation.	Solution.
y must be exterminated in this case; and giving to this unknown quantity the same coefficient in both equations, we added them to one another, by which a new equation was formed, from whence was deduced the value of x. We have the value of y immediately from the second equation.	$anx - bny = nd$ $bny = bp$ $anx = nd + bp$ $x = \dfrac{nd + bp}{an}$ and $y = \dfrac{p}{n}$

N. B. We could have the value of x by substituting into the first equation, the value of y deduced from the second, but this way is not shorter than that which we have used, and in a case where this value of y would be complicated, we would certainly find this process longer.

Problem. *Two persons owe 29£. they both have money, but not so much that either of them would be*

able to pay the common debt by herself; if the first had the $\frac{2}{3}$ of the money of the second, she alone would be able to pay the debt; and, if the other had the $\frac{3}{4}$ of the money of the first, she also would be able to pay the debt alone. We demand how much money is possessed by each?

Let x be the money of the first person, and y that of the second; and now let us examine the question. It is necessary, in order that the problem may be soluble, to find in it two equations, since there are two unknown quantities; but these two equations are easily perceived; for since the money of the first person is x and that of the second y, we shall evidently have - - - - - - - - - - $x + \frac{2}{3}y = 29$

and $y + \frac{3}{4}x = 29$

EXPLANATION.

We began by making the denominators disappear; then, as one of the unknown quantities is found to have the same coefficient in both equations, it has been exterminated by subtracting the first equation from the second; from whence we easily deduced the value of y.

We substituted this value of y into the second equation of the problem, which being solved, gave the value of x; we have proceeded step by step, that we may the better shew the method they ought to follow.

SOLUTION.

$$3x + 2y = 87$$
$$4y + 3x = 116$$

$$2y = 29$$
$$y = 14\tfrac{1}{2}$$

$$14\tfrac{1}{2} + \frac{3x}{4} = 29$$

$$\frac{29}{2} + \frac{3x}{4} = 29$$

$$\frac{58 + 3x}{4} = 29$$

$$58 + 3x = 116$$
$$3x = 116 - 58$$

$$x = \frac{58}{3} = 19\tfrac{1}{3}$$

Thus the first person had $19\frac{1}{3}$ £, *and the second* $14\frac{1}{3}$ £. For the proof, it is necessary to substitute these values of x and y into the first equation, which ought to be identical if the work has been done right.

We shall not give a general example for the extermination of unknown quantities, when there are three equations; this calculation is long and complicated; the solution of a problem will suffice for clearly comprehending the method to be followed.

PROBLEM. *A captain wishing to attack an intrenchment with three companies, the one of grenadiers, the other of riflemen, and the third of fusileers, promises to his troop a recompence of* 901 *dollars, to be distributed as follows. Every soldier of the company that shall first force the intrenchment, will receive one dollar, and the remainder of the money will be equally distributed between the two other companies.*

It is found, that if the grenadiers force the intrenchment first, each soldier of the two other companies, will receive half a dollar; but if it be the riflemen, each one of the others will receive $\frac{1}{3}$ *of a dollar, and if it be the fusileers, the grenadiers and riflemen will each receive* $\frac{1}{4}$ *of a dollar. It is required to tell how many soldiers the captain had?*

Though by the demand, it seems that the problem contains but one unknown quantity, it is evident, from a little reflection, that in reality it contains three, because, to have the total number of soldiers, it is necessary to know the number of each company in particular.

Thus let us call x the number of soldiers of the company of grenadiers, y that of the riflemen, and z that of the fusileers. Now, then, it is

necessary to find three equations in the enunciation of the problem; They are easily perceived.

For, if it be the grenadiers that force the intrenchment first, we shall have the first equation $x + \frac{y+z}{2} = 901$; if it be the riflemen, we shall have the second equation $y + \frac{x+z}{3} = 901$; if it be the fusileers, we shall have the third equation $z + \frac{x+y}{4} = 901$.

Let us begin by clearing off the denominator, which is found in each of the equations, and let us suppose $901 = a$. (It is always proper for facilitating the calculation, to make such suppositions, when we have, in the equation numbers rather large.) These three equations will become

1st $2x + y + z = 2a$

2nd $3y + x + z = 3a$

3d $4z + x + y = 4a$

EXPLANATION.

By subtracting the first equation from the second, we exterminated z, which gave the fourth equation; then to have another equation between x, and y, we compared the first equation with the second, and multiplied all the terms of the first by 4, the coefficient of z in the third, which gave the fifth equation, from

SOLUTION.

4th $2y - x = a$

5th $8x + 4y + 4z = 8a$

6th $7x + 3y = 4a$

7th $14y - 7x = 7a$

$17y = 11a$

$$y = \frac{11a}{17}$$

which we substracted the third in order to have a new equation between x and y. Then we compared the fourth and sixth, and to exterminate x we multiplied all the terms of the fourth by 7 the coefficient of x in the sixth, after which we added this to

and putting in the place of a its value

$$y = 583$$
$$2 \times 583 - x = 901$$
$$x = 1166 - 901$$
$$x = 265$$

$$2 \times 265 + 583 + z = 1802$$
$$z = 689$$

the sixth; we had then an equation from which was deduced the value of y. This value of y was substituted into the fourth equation, and having performed the calculation, the value of x was found. Finally, the values of x and y were put into the first equation, which gave the value of z. *The Captain, then, had 265 grenadiers, 583 riflemen, and 689 fusileers, which makes the total number of soldiers 1537.*

For the proof, it would be necessary to substitute the values of the three unknown quantities into the second or third equation of the problem, which ought to come out identical, the calculation having been performed right.

If a problem had four unknown quantities and four equations, it would be necessary, according to the same method, to compare these equations two and two, in such a manner as to have no more than three equations and three unknown quantities. It is plain, that the work for being longer, would not be more difficult, to solve a problem which should contain any number of equations.

INDETERMINATE PROBLEMS

OF THE FIRST DEGREE.

Indeterminate problems are capable of many solutions; but, as their number is generally limited, by certain conditions expressed or understood in the problem, the researches, which have been made by Algebra in questions of this kind, cannot but be curious and interesting; we are going to propose a single example of them, which will suffice to give an idea of the considerations to be made, to determine the number of solutions, which can answer to the same question.

PROBLEM. *To make up 360 dollars with 22 pieces of 3 different coins; the first of 24d, the second of 12d, and the third of 6d.*

Let x, y, z, be respectively the numbers of the pieces of 24d, of 12d, and of 6d. We shall have first, the equation $x + y + z = 22$. Farther, it is evident, that the number x of pieces of 24d, gives $24x$ pence; that y of the pieces of 12d, gives $12y$ pence; and lastly, that z of the pieces of 6d, gives $6z$ pence. Now the sum of these three numbers of pence, ought to be 360. We have, then, this second equation $24x + 12y + 6z = 360$. As all the conditions of the question are expressed, and as we have three unknown quantities, and but two equations, we cannot come immediately at the determinate values of x, y, z. But if we give one of the three unknown quantities, the others will necessarily be determinate.

In effect, taking for example the value of z in the first equation, we shall have $z = 22 - x - y$; and substituting this value into the second equation, it will

become $24x + 12y + 132 - 6x - 6y = 360$; from which is deduced $y = 38 - 3x$. Putting this value of y into the equation $z = 22 - x - y$, we shall find $z = 2x - 16$; whence it follows, that, by giving different values to x, we shall have the correspondent values of y and z.

Now, according to the conditions of the question, the numbers x, y, z ought to be *whole*, since there cannot enter either halves or quarters into the sum 360*d*: Farther, that these same numbers may be positive, it is necessary that $38 - 3x$, and $2x - 16$, should not be less than 1. The first condition demands that x, which ought to be a *whole*, should not exceed 12; and the second, that x should not be less than 8.

The values which can be given to x, then, are comprehended between the limits 12 and 8. The values of y and z are found, by substituting in the above equations the different values of x.

For example, let $x = 12$, we shall have $y = 2$ and $z = 8$.

Let $x = 11$, we shall have $y = 5$ and $z = 6$

Let $x = 10$, we shall have $y = 8$ and $z = 4$

Let $x = 9$, we shall have $y = 11$ and $z = 2$

Let $x = 8$, we shall have $y = 14$ and $z = 0$

The last solution should be rejected, because it excludes the parts of 6d. If it should be admitted, we should have but pieces of 24d. and 12d, to form the sum 360.

Thus the sum 360*d*. being composed of pieces of 24d, of 12d, and of 6d, the problem has but four solutions.

These same solutions could be found, by beginning to give determinate values to y and z, instead of beginning by x; for it is evident, that being conducted by the same considerations, we ought to have the same results.

EQUATIONS OF THE SECOND DEGREE.

$X^2 + p x + q = 0$, *is the most general equation of the second degree*; for, whatever be the number of terms that can be supposed multiplied by x^2, we shall always be able to form them all into one term, which shall have for its coefficient the sum of all the particular coefficients of x^2, and then dividing all the terms of the equation by this single coefficient, x^2 will be found freed from every coefficient. In the same way we can form a single coefficient of all the particular coefficients of the terms, multiplied by x, and suppose this single coefficient equal p; finally, we can represent by q all the terms in which the unknown quantity is not found.

To solve this general equation in all cases which may offer, let us make different suppositions with respect to p and q, and let us first suppose $p = 0$.

The equation will then be $x^2 + q = 0$, and $x^2 = -q$; we shall have, by extracting the square root of both members, $x = \pm\sqrt{-q}$; thus this particular case offers no difficulty, we can easily find the value of x. Before proceeding farther, we will here make two observations.

The first relates to the double sign before the radical; we should always put this double sign before a square root, because $+ \times +$, and $- \times -$ equally

give $+$ for the product; *and thus we cannot determine whether a given square comes from a positive or negative root.*

We do not put the double sign before the first member of the equation, because that would be useless, since we could have but two values for x, though it should appear as if we could have four values of it, by the combination of the double signs. In effect, then, let $\pm x = \pm \sqrt{-q}$; as we cannot suppose both signs at the same time, we shall have, therefore, on one side, $+x = \pm \sqrt{-q}$, and on the other, $-x = \pm \sqrt{-q}$; from each of these two equations we shall draw two values for x, on account of the double sign before the radical; the 1st. will give $\begin{cases} +x = +\sqrt{-q} \\ +x = -\sqrt{-q} \end{cases}$, the 2nd will give $\begin{cases} -x = +\sqrt{-q} \\ -x = -\sqrt{-q} \end{cases}$. If we change the signs in these two last values of x, we shall have $\left\{\begin{matrix} +x = -\sqrt{-q} \\ +x = +\sqrt{-q} \end{matrix}\right\}$ values, exactly the same with those that the first equation gives, *therefore it is useless to put the double sign before the first member of the equation.*

The 2nd observation relates to the sign of the quantity, which is under the radical. The equation $x = \pm \sqrt{-q}$ will give an imaginary value for x, as long as q shall be by itself a positive quantity; because $-q$ will then necessarily be a negative quantity, and this quantity representing a square, the question must contain some absurdity, *since we cannot have the root of a square preceded by a negative sign, as every root, whether positive or negative, always gives a positive square.*

Let us continue the examination of the general equation, and let us now see, if there is any difficulty in treating this equation, when $q = 0$. We shall then have $x^2 + p x = 0$, dividing all the terms by x, we shall have $x + p = 0$, and $x = -p$.

These two particular cases being examined, it remains, then, for us to solve the equation $x^2 + px + q = 0$, in all its general forms.

It is necessary to begin, by putting the equation under this form $x^2 + p x = -q$; *placing all the terms combined with x, into the first member of the equation.* Then we remark, that, if the first member of the equation was a perfect square, it would be easy to reduce this equation to an equation of the 1st degree, by taking the square root of both members of the equation.

Now we know, *that the square of a binomial is composed of the square of the first term of the binomial, of twice the second by the first, and of the square of the second.* But we can consider $x^2 + p x$ as the two first terms of the square of a binomial, of which x would be the first term of the root, and then p would be double of the second; thus, to complete the square, nothing more would be necessary, than to add to it the square of the second term of the root, that is to say, the half of p raised to a square; but to preserve the equality, it is necessary to add, in both members of the equation, the same quantity; thus our equation will become then $x^2 + p x + \frac{p^2}{4} = \frac{p^2}{4} - q$; taking the square root of each member, we have $x + \frac{p}{2} = \pm \sqrt{\frac{p^2}{4} - q}$; and finally $x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$, *the general expression of the value of x, for every equation of the second degree.*

Thus, after having put a problem into an equation, we shall have immediately the value of x, by substituting into this expression, in place of p and q, the quantities which they represent.

If the term px had been negative, the second term of the root $\frac{p}{2}$ should also be negative; we have told the reason of it under the head of Involution.

It is necessary to remark, that the solution which is obtained, always gives two values for x, because of the double sign which is found before the radica part; one of the values is $x = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}$, and the other $x = -\frac{p^2}{4} - \sqrt{\frac{p^2}{4} - q}$; and as these two values are unequal, it is evident they cannot answer to the same question; but it is no less true, that both one and the other are satisfactory to the equation $x^2 + px + q = 0$; that is to say, that by substituting either value of x into the equation, it will be identical, all the terms of the 1st member will be reduced to zero.

From these two different values for x we shall conclude, *that problems, which lead to an equation of the second degree, always contain in the combinations of known with unknown quantities, two kinds of questions, which have for solution either of the values of x.*

Let us resume the equation $x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$. I see that the value of x will always be real, when the quantity q shall be by itself a negative quantity, because then $-q$ will be positive, and $\frac{p^2}{4}$ being always positive whether $\frac{p}{2}$ be positive or negative, the quantity under the radical will therefore be always a positive quantity. The values of x will be imaginary only

when q shall be a positive quantity, and greater than $\frac{p^2}{4}$.

Let us apply to an example the general expression which we have found of the value of x.

First Problem. *To find a number, the square of which, less the third of the same number, be equal to 8.*

Let x be this number, the equation will be $x^2 - \frac{x}{3} = 8$, or $x^2 - \frac{x}{3} - 8 = 0$. Comparing this equation with the general equation, we have $p = -\frac{1}{3}$ and $q = -8$; substituting these values of p and q into the general formula $x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$, we shall have $x = \frac{1}{6} \pm \sqrt{\frac{1}{36} + 8} = \frac{1}{6} \pm \sqrt{\frac{289}{36}} = \frac{1}{6} \pm \frac{17}{6}$. From thence we shall draw two values for x; the first $x = 3$, and the second $x = -\frac{8}{3}$.

It appears as if none but the first value could satisfy the question, which would be true, if we suppose that the number demanded was positive; but as the enunciation of the problem, can equally agree with the supposition of a negative number, the second value of x relates to this last case; and if in the equation of the problem, we put successively in place of x its two values, this equation will be identical. For, in the first place we shall have $9 - 1 = 8$, and in the last place $\frac{64}{9} + \frac{8}{9} = 8$.

When we have not the general formula before us, and cannot remember it, we can solve the equation of the problem, by following the method employed to solve the general equation; this extends but little the

calculation. We are going to exercise beginners in solving the same problem by this method.

I take again the equation $x^2 - \frac{x}{3} = 8$. We ought first to complete the square of the first member by adding to it half of the coefficient of the second term raised to a square; now this coefficient is here $\frac{1}{6}$. Thus we shall have $x^2 - \frac{x}{3} + \frac{1}{36} = \frac{1}{36} + 8$; and taking the square root of both members, there results $x - \frac{1}{6} = \pm\sqrt{\frac{1}{36} + 8}$; from whence we have $x = \frac{1}{6} \pm \frac{17}{6}$.

Second Problem. *A reservoir full of water has three orifices A, B, C; it can empty itself by the three orifices together, in 6 hours; by the orifice B alone, it would empty itself in three-fourths of the time that it would take to empty itself by A alone; and by C, in a space of time, which is greater by 5 hours than the time by B. We ask in what time the reservoir will empty by each of these orifices separately?*

The rapidity with which the water runs out is supposed uniform, and always the same in all cases.

If we knew the number of hours, in which the reservoir could empty itself by the orifice A alone, we should know the time it would take, to empty itself by each of the other orifices, thus let us call this number of hours x. Now let us study the question, and try to find an equation.

According to the enunciation of the question, the reservoir can empty itself by the three orifices in 6 hours; consequently if we can express the quantity of water, which has run out through each orifice in 6

hours, the sum of these three quantities ought to equal the capacity of the reservoir; let us call 1 this capacity, or the whole of the water that is contained in the reservoir.

Since we suppose, that all the water would escape by the orifice A alone in x hours, we shall say; if in x hours all the water of the reservoir runs out, how much will run out in 6 hours; that is to say we have this proportion, $x : 1 :: 6$: a fourth term equal $\frac{6}{x}$, which expresses the quantity of water that ran out in 6 hours by the orifice A alone.

We shall see by a similar reasoning, that the quantity of water escaped by B alone $= \frac{6}{\frac{3}{4}x}$; and by C alone $= \frac{6}{\frac{3}{4}x+5}$: thus we shall have the equation $\frac{6}{x} + \frac{6}{\frac{3}{4}.x} + \frac{6}{\frac{3}{4}.x+5} = 1$, or $6\left(\frac{1}{x} + \frac{1}{\frac{3}{4}x} + \frac{1}{\frac{3}{4}x+5}\right) = 1$, or $6\left(\frac{1}{x} + \frac{4}{3x} + \frac{4}{3x+20}\right) = 1$, or $6\left(\frac{7}{3x} + \frac{4}{3x+20}\right) = 1$; clearing away the fractions, and reducing, we shall find $x^2 - \frac{46}{3}x = \frac{280}{3}$. Adding to both sides the square of $\frac{23}{3}$, we shall have $x^2 - \frac{46x}{3} + \frac{529}{9} = \frac{529}{9} + \frac{280}{3}$. Extracting the square root from both members, we shall have $x - \frac{23}{3} = \pm \frac{37}{3}$; and $x = \frac{23}{3} \pm \frac{37}{3}$; that is to say, $x = 20$, or $x = -\frac{14}{3}$. We see

plainly, that the first value only will answer to the enunciation of the problem. *Thus the three sought times will be, 20 hours for the orifice A, 15 for the orifice B, and 20 for the orifice C.*

The proof will be seen by putting for x its value in the first equation.

Equations of the second degree are not the only ones that can be solved by the general method that we have just given; such as $x^{2m} + px^m + q = 0$, are also in the same case; *those equations never contain but two terms, in which the unknown quantity is found, but with different exponents, and such, that the higher of these exponents is precisely double of the smaller.*

To bring the proposed equation to an equation of the second degree, let us make $x^m = z$; we shall have $x^{2m} = z^2$, and then the equation will be $z^2 + pz + q = 0$;

from which we deduce $z = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$, which

gives then $x^m = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$; and finally,

$$x = \sqrt[m]{-\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}}.$$

Whatever be the value of m, we shall always find the expression of the radical, either by the direct way, or the binomial rule of Newton, or yet more simply by the assistance of logarithms, as we shall see farther on.

EQUATIONS OF ANY DEGREE.

There is no general method for solving equations of the 5th degree and above; we have formulas for the

solution of equations of the 3d and 4th degree, but they have the inconvenience of not always giving *the root of the equation*; or the values of the unknown quantity under a limited form.

The English have invented a very simple way for solving equations of all degrees, *when with the unknown quantity, these equations contain none but numerical quantities*; it is, in reality, a mere boggling, but we do not obtain by it results, less exact than by any other way, and the calculation of the highest exponents becomes easy by the assistance of logarithms.

This way consists in attributing different values to the unknown quantity, and substituting them successively into the equation, until we find some, which shall make the equation identical; there will be, *as we have seen, for the confirmation of calculations in equations of the first and second degree*, the only true ones, and which can answer the state of the question, which shall have produced this equation.

It is necessary to know, besides, that there are always in every equation, as many values for the unknown quantity, as there are unities in the degree of the equation. We are going to apply to an example the way to be followed.

Let the equation be $x^3+5x^2-44x=-60$.

I try at first to make $x=1$; then the equation becomes $1+5-44=-60$, *an absurd result*, therefore my supposition is false. Then I suppose $x=2$, and substituting, we have $8+20-88=-60$, *an identical equation*, from which I conclude *that 2 is one value of x*.

To find the two other values, I continue my suppositions, and I make $x = 3$; the equation becomes $27 + 45 - 132 = -60$, an identical equation; 3 *then is another value of* x; I suppose $x = 4$: and then I have $64 + 80 - 176 = -60$, *an absurd equation*; from which I conclude, that 4 is not one of the values of x, and I easily see, that by supposing for x values which go on augmenting continually, I shall find results still more erroneous; it is evident then, I cannot look for the third value of x, but among negative quantities. Effectively we shall find after some trials, that *this value is* $x = -10$, since, when substituted into the equation, it renders it identical; for we have $-1000 + 500 + 440 = -60$.

RATIOS AND PROPORTIONS.

We cannot compare two quantities together but in two ways; the first by considering how much one surpasses, or is surpassed by the other, and the second by considering how often one contains or is contained by the other. The first way of comparing is called *Arithmetic ratio*, and the second *Geometric ratio*, or simply *ratio*. Two terms, then, are necessary to express a ratio; that, which we name the first, is called *the Antecedent*, and the other *the Consequent*.

A *proportion* is the equality of two ratios; there are then two sorts of *proportion*, the one *Arithmetic*, the other *Geometric*. Thus every proportion is composed of four terms, two *antecedents* and two *consequents*; we distinguish them also by *extremes* and *means*.

We have given in arithmetic a sufficient detail of these general notions.

ARITHMETIC PROPORTIONS.

In every arithmetic proportion, the sum of the extremes is equal to that of the means.

Let the proportion be $a . b : m . n$, then I say $a + n = b + m$; for, since there is a proportion, the ratio of a to b is equal to that of m to n; from whence it follows, that b surpasses, or is surpassed by a, by the same quantity, by which n surpasses or is surpassed by m; let us name d this quantity; we shall have $b = a \pm d$, and $n = m \pm d$; substituting these values of b and n into the proportion, we shall have $a . a \pm d : m . m \pm d$, *which will be*, as we see, *the general expression of every arithmetic proportion*; but it is evident that the sum of the extremes is equal to that of the means, since those two sums are composed of the same letters; therefore in every particular proportion also, the sum of the extremes is equal to that of the means; *therefore* $a + n = b + m$, which was to be demonstrated.

From that follows an easy way for finding the fourth term of an arithmetic proportion, when we know the other three; for if it be an extreme, it will be evidently equal to the sum of the two means less the other extreme; and if it be a mean, it will be equal to the sum of the two extremes less the other mean.

We call *continual*, all such proportions as $a . b : b . c$ where the two means are equal: we often write it thus $\div a . b . c$, and call the middle term *an arithmetic mean proportional.*

It is evident, that, in an arithmetic continual proportion, *the middle term*, or *mean proportional, is equal to half the sum of the two extremes.*

We can change the places of the terms of an arithmetic proportion, without destroying it, provided that the sum of the two extremes be always equal to that of the means. We can also add, or subtract equal quantities from two terms of a proportion without destroying it, provided we work at the same time on an extreme and a mean, since then the sum of the extremes always remains equal to that of the means.

GEOMETRICAL PROPORTIONS.

In every Geometric proportion the product of the extremes is equal to that of the means.

Let the proportion be $a : b :: c : d$, I say, then, $a \times d = b \times c$; for, since there is a proportion, the ratio of a to b is equal to that of c to d; from which it follows that b contains a as often as d contains c, (the reasoning would be the same if we suppose, that a contains b a certain number of times); let us name q this number of times; we shall have $b = a\,q$, and $d = c\,q$; putting these values into the proportion, we shall have $a : a\,q :: c : c\,q$, *which will be the general expression for every geometric proportion*; but it is evident, that in this general proportion, the product of the extremes is equal to that of the means, since these two products are composed of the same factors; therefore in every particular proportion the product, of the extremes is equal to that of the means; *therefore* $a \times d = b \times c$, which was to have been demonstrated.

From that follows the way to find the fourth term of a geometric proportion, the three first terms being known; for if it be an extreme, it is equal to the product of the two means divided by the other extreme, and if it be a mean, it is equal to the product of the two extremes divided by the other mean.

In a continual proportion such as $a : b :: b : c$, which is often expressed thus $\div\!\div\ a : b : c$, *the middle term*, or *the mean proportional, is equal to the square root of the product of the two extremes*. From this, *that, every proportion gives an equation*, it follows, *that we can always put an equation into a proportion*; let us take an example which may appear embarrassing, let us suppose $mn = p$. I shall put this equation into a proportion as follows $m : p :: 1 : n$.

We can multiply or divide by the same quantity, two terms of a geometric proportion without destroying it, provided we work at the same time on a mean and an extreme; because then the sum of the extremes will remain equal to that of the means. *We can also make every transposition of terms that shall be required*, under the same condition, that the product of the extremes be always equal to that of the means.

If we multiply two geometric proportions, term by term, the four products will form a proportion. Let the two proportions be $\left\{\begin{matrix} a : b :: c : d \\ h : k :: m : n \end{matrix}\right\}$ then I say, that if these two proportions be multiplied together term by term, we shall have this new proportion $ah : bk :: cm : dn$; for, from the first proportion we deduce $ad = bc$, and from the second $hn = km$; multiplying these two equations, member by member, we shall have $adhn = bckm$, from which is deduced the proportion, which was to be proved, $ah : bk :: cm : dn$.

From this, it is necessarily concluded that if four quantities are in proportion, all the like powers of these same quantities will be also in proportion; for, if for example, $a : b :: c : d$, multiplying this proportion by itself, term by term, we shall have then also $a^2 : b^2 :: c^2 : d^2$,

and if we multiply this by the first, we shall have again $a^3 : b^3 :: c^3 : d^3$, thus in succession; and *vice versa, if four powers are in proportion, their like roots will also be in proportion.*

We call a *compound ratio*, the ratio of any proportion formed of the product of several other proportions multiplied term by term; if the ratios of those different proportions be equal, the ratios are called *double, triple, &c. ratios*, according to the number of proportions that have been multiplied.

If there is a succession of equal ratios, such as $a : b :: c : d :: e : f :: g : h ::$ &c. I say, then, that we shall always have this proportion, *the sum of the antecedents is to that of the consequents, as any one antecedent is to its consequent*, that is to say, then $a + c + e + g$ &c. $: b + d + f + h$ &c. $:: a : b$.

To demonstrate it, let us consider at first, none but the proportion $a : b :: c : d$; making an *alternando*, that is to say, changing the places of the means, I write it thus, $a : c :: b : d$; now let us suppose that a is greater than c (if on the contrary c was greater than a, the reasoning would be the same,) it is clear that since there is proportion, a will contain c as often as b will contain d, and consequently *componendo*, $a + c$ will contain c, as often as $b + d$ will contain d; we shall have then the proportion $a + c : c :: b + d : d$, and again making an *alternando*, this proportion will become $a + c : b + d :: c : d$; now returning to the proportion $a : b :: c : d$, we see demonstrated *that the sum of the antecedents of this proportion is to that of the consequents, as an antecedent is to its consequent*; but, as the reasoning we have just made, is not confined particularly to the quantities a, b, c, d, it follows, therefore, that *this truth is*

applicable to every geometric proportion; this being granted, it is easy to prove the proposition in question. For, since $a+c : b+d :: c : d$, and as the succession of equal ratios gives us $c:d::e:f$, it follows, that $a+c:b+d::e:f$; but from this proportion we deduce $a+c+e:b+d+f::e:f$; now from the succession of equal ratios we have again $e:f::g:h$, therefore we shall also have $a+c+e:b+d+f::g:h$, and consequently $a+c+e+g:b+d+f+h::g:h$ or $::a:b$. It is evident, that this reasoning demonstrates the proposition advanced for an infinite series of equal ratios.

We shall remark, that in gaining our end, it has been demonstrated, that in a series of equal ratios, *any one sum of antecedents is to a like sum of consequents, as any one antecedent is to its consequent.*

Let us resume the proportion $a:b::c:d$; it is evident, by a demonstration exactly similar to that of the *componendo*, that we shall have *detrahendo*, $a-c:b-d::a:b$; that is to say, that *in every proportion, the difference of the antecedents, is to the difference of the consequents, as an antecedent is to its consequent.*

ARITHMETIC PROGRESSIONS.

An arithmetic progression is a series of terms, among which reigns a like difference.

Thus, every arithmetic progression can be expressed by this one $\div a . a \pm d . a \pm 2d . a \pm 3d - - - - . x$ supposed the last term; for all those terms have among them the same difference, which is d. The progression will be *increasing* or *decreasing*, *according as the difference* (or the ratio) *d shall be positive or negative.*

We see, that any one term of this series is equal to the first, more or less the difference, taken as often as there are terms before it. Thus *the last term of a progression is therefore equal to the first term, more or less the difference, taken as often as the number of terms, less one.*

If then we name n the number of terms in the progression, we shall have $x = a \pm d \times \overline{n-1}$.

Let us seek now the expression of the sum of this same general progression; but to facilitate this enquiry, let us suppose for a moment, that the progression consists but of four terms, and that it goes on increasing; what we shall say, will be applicable to a progression of any number of terms, and decreasing.

$$
\begin{array}{lllll}
\text{Let it be} & \div a. & a+d. & a+2d. & a+3d \\
 & \div a+3d. & a+2d. & a+d. & a \\
\hline
 & 2a+3d & 2a+3d & 2a+3d & 2a+3d
\end{array}
$$

I write below this same progression, but reversed, in such a manner that the last term is found under the first; I add then these two progressions, term by term, *which gives as many particular sums as there are terms in the proposed progression.* But we shall remark, that all these sums are equal, and that to have their total value, it will be sufficient to multiply one of them by their number, that is to say, *by the number of terms in the progression*; but we see, that the first sum for example, is equal to the sum of the first and last term of the progression, therefore it follows that *their total value will be equal to the sum of the first and last term of the progression, multiplied by the number of terms of*

the same progression: But the total value of all these particular sums, is evidently the double of the value, or of the sum, of the progression; therefore if we name S this sum, we shall have, by resuming the general denominations employed first, $2\,S = \overline{a+x}\,.\,n$, and $S = \overline{a+x}\,.\,\frac{n}{2}$. That is to say, then, *that the sum of the terms of any arithmetic progression, is equal to the sum of the first and last term of the progression, multiplied by half the number of terms in the same progression.*

By means of the two general equations $x = a \pm d\,.\,\overline{n-1}$ and $S = \overline{a+x}\,\frac{n}{2}$, *when we shall know three of the five quantities which constitute an arithmetic progression, we shall always be able, according to the laws of Analysis, to find the two others.* These five constituent parts of a progression, are, as we see, *the first term, the last term, the number of terms, the difference, and the sum.*

Every problem, which shall give three of these five quantities, will be always easy to solve, as we shall see immediately by some examples.

If, in an arithmetic progression, we take four such terms that there be, as many terms of the progression between the first and second, as between the third and the fourth, these four terms will form an arithmetic proportion, since the difference of the two first terms will be equal to that of the two last, as, both the one and the other of these differences, are evidently formed of the difference of the progression, taken the same number of times; thus, for example, we see, that $a\,.\,a \pm 2d : a \pm d\,.\,a \pm 3d$.

First Problem. *To insert between 3 and 15, five arithmetic proportional means.*

That is to say, to find five terms forming with 3 and 15, an arithmetic progression, of which these two numbers shall be the extremes.

If we knew the difference of the progression, it would be easy to find the required terms; but it is easy to have this difference, for, *we here know three of the five quantities which constitute the progression,* viz. the first term 3, the last term 15, and the number of terms which is 7. Thus, to make use of the general equation $x = a + d \cdot \overline{n-1}$, we deduce from it $d = \frac{x-a}{n-1}$, and whereas we have $a = 3$, $x = 15$, and $n = 7$; substituting in place of these letters, their values, we shall have $d = \frac{15-3}{6} = 2$; then the progression will be $\div 3 . 5 . 7 . 9 . 11 . 13 . 15$, which gives the five proportional means required.

Second Problem. *A preceptor tells his pupil, that the first punishment he should deserve, he would give him* 10 *verses to learn by heart, the second* 13, *the third* 16, *and so on. It is found at the end of the year, that the child had learnt* 4175 *verses. It is required to tell how many times he was punished,* and *how many verses he learnt on his last punishment.*

The question evidently presents an arithmetic progression, of which 10 is the first term, 3 the differ-

ence, and 4175 the sum of the terms; and it is not less evident, that the number of times that the child has been punished, gives the number of terms in the progression, and that the number of verses, which he was obliged to learn the last time he was punished, forms the last term of the progression.

Thus we see, then, that of the *five quantities which constitute a progression, three are known*; now, if we apply the progression, which this question gives, to the general progression, we shall have $a = 10$, $d = 3$, $S = 4175$; then nothing more is necessary, than to find the values of n and x, by means of the two general equations $x = a + d \,.\, \overline{n-1}$, and $S = \overline{a + x} \,.\, \frac{n}{2}$, which we shall easily deduce by the laws of analysis.

Let us substitute into the second equation the value of x, which is given to us by the first; we shall have $S = (a + a + d \,.\, \overline{n-1})\frac{n}{2}$, and performing the multiplications; $2S = 2an + dn^2 - dn$; n being here the unknown quantity, it is an equation of the second degree that we have to solve; we put it at first under this form, $n^2 + \left(\frac{2a-d}{d}\right)n = \frac{2S}{d}$, from which we deduce, according to the last given rules for the solution of equations of the second degree, $n = -\frac{2a-d}{2d} \pm \sqrt{\left(\left(\frac{2a-d}{2d}\right)^2 + \frac{2S}{d}\right)}$; now, substituting in place of a, d and S, their values, we shall have $n = -\frac{17}{6} \pm \frac{317}{6}$.

Of these two values of n, the one positive, and the other negative; it is the first only, which can answer to the question, and which gives $n = 50$. Now, substituting into the equation $x = a + d \cdot \overline{n-1}$ in place of a, d, n, their values, we shall have $x = 157$.

Thus the child was punished 50 *times; and, on the last punishment, he learnt* 157 *verses.*

Third Problem. *A horse at full speed, runs* $1144\frac{1}{2}$ *feet the first minute,* $1141\frac{1}{2}$ *the second,* $1138\frac{1}{2}$ *the third, and so on. We ask how many miles he shall have run in one hour.*

That is to say, the question is, to find the whole road run over in 60 minutes. But this whole road is equal to the sum of the different spaces run over in each minute of the course, and we see that these different spaces form a decreasing arithmetic progression, of which $1144\frac{1}{2}$ is the first term, 3 the difference, and 60 *the number of terms*, since every minute there is a new space run over.

Now, applying this progression to the general progression, we have $a = 1144\frac{1}{2}$, $d = 3$, $n = 60$, and then the question is to find S: but we have $S = \overline{a+x}\,\frac{n}{2}$; now this equation is not sufficient to determine the value of S, because x is not known; it is necessary to have recourse to the other general equation $x = a - d \cdot \overline{n-1}$; substituting this value of x, we shall have $S = (2a - d \cdot \overline{n-1}\,\frac{n}{2})$; now, putting in place of a, d and n, their values, we shall find $S = 63360$ feet, which make 10560 fathoms.

Thus then, the horse will have run 10560 fathoms, or 12 miles in one hour.

It has no doubt been remarked, that in the use we have made, of the formula $x = a \pm d \, . \, \overline{n-1}$, we have employed the sign + only, in the solution of the two first questions, because *the progressions were increasing*, and the sign — only in the last, because *the progression was decreasing*.

These examples are quite sufficient, to shew, *how by the aid of the two general equations*, it is possible to solve all questions dependent on Arithmetic progressions.

GEOMETRIC PROGRESSIONS,

A geometric progression, is a series of terms so graduated, that the division of any term, by the one immediately preceding, always gives the same quotient.

Thus every geometric progression can be represented by this $\div\!\div\, a : aq : aq^2 : aq^3 : aq^4 \text{ - - - } : x$ last term. *The progression will be increasing when q shall be greater than* 1, *and it will be decreasing when q shall be less than* 1.

It is clear, that any one term is equal to the first multiplied by the ratio, raised to a power equal to the number of terms before it. Thus then *the last term will be equal to the first multiplied by the ratio, raised to a power equal to the number of the terms of the progression, less one.* Let us name n the number of terms, we shall have then $x = aq^{n-1}$.

To find the expression of the sum which we shall name S, it is necessary to remark, that every geomet-

ric progression furnishes a series of such equal ratios, that the *consequent* of any one ratio serves as *antecedent* to the ratio which succeeds it.

In effect, we evidently have $a : aq :: aq : aq^2 :: aq^2 : aq^3 :: aq^3 : aq^4 ::$ &c. but in this series of equal ratios, *all the terms of the progression are antecedents except the last, and all consequents except the first*; thus, by recollecting, that in a series of equal ratios, the sum of the antecedents is to that of the consequents, as any one antecedent to its consequent, we have $S - x : S - a :: a : aq$, or $:: 1 : q$; then, making the product of the extremes equal to that of the means, we find $qS - qx = S - a$, from which is deduced

$$S = \frac{qx - a}{q - 1}.$$

By means of this equation, and of that $x = aq^{n-1}$ *when we shall know three of the five quantities which constitute a geometric progression, we shall always be able to find the two others*, by the rules of analysis, and by the help of logarithms, of which we shall speak in the succeeding article.

In every geometric proportion we have these different proportions, *the square of the first term is to the square of the second, as the first term is to the third*; ——*the cube of the first term is to the cube of the second, as the first term is to the fourth, and so on*; that is to say, that $a^2 : a^2q^2 :: a : aq^2$——that $a^3 : a^3q^3 :: a : aq^3$——that &c. which is demonstrated by the reason, that the product of the extremes is found equal to that of the means.

If in a geometric progression, four such terms are taken, that there shall be as many terms of the progression, between the first and the second, as between the third and the fourth, these four terms will form a geometric proportion, since the ratio of the two first terms will be equal to that of the two last, as, both the one and the other of these ratios, are evidently composed of the ratio of the progression, raised to the same power; thus we see by example, that $a : aq^3 :: aq : aq^4$.

We are going to make the application of the two formulæ which we have found, to some problems.

First Problem. *To insert between 2 and 54, two geometric proportional means.*

That is to say, find two terms forming with 2 and 54 a geometric progression, of which these two numbers shall be the extremes.

To solve the question, nothing more is required than to find the ratio of the progression. Now, the first term 2, the last 54, and the number of terms of this progression, which is four, are known; thus, to apply this progression to the general progression, we have $a = 2$, $x = 54$, and $n = 4$; substituting these values into the equation $x = aq^{n-1}$, we have $54 = 2q^3$, and $q = \sqrt[3]{27} = 3$; then the progression will be $\div 2 : 6 : 18 : 54$, *thus* 6 *and* 18 *are the two proportional means required.*

If the number of proportional means had been greater, 6 for example, we should have had $q^7 = 27$ and $q = \sqrt[7]{27}$; but to have, by an easy process, a like root, it is neceesary to have recourse to *logarithms.*

Second Problem. *3 being the first term of a geometric progression, 5 the ratio, and 46875 the last term; we demand the number of the terms of the progression?*

This problem can be solved by developing the progression, since we know the first term and the ratio; but this way would be very long, if there were a very great number of terms. Here is a method more direct; the equation $x = aq^{n-1}$ becomes, by substituting for x, a, and q, their values, drawn from the state of the question, $46875 = 3 \times 5^{n-1}$; but now to disengage n and to have its value, it would be necessary to have recourse to logarithms; it will be found equal to 7.

Third Problem. *It is required to find the sum of this series, continued on to infinity,* $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \&c.$

It is obvious that all these numbers form a decreasing geometric progression, and their sum is to be found.

Now we know here the first term of the progression, which is $\frac{1}{2}$, the ratio which is also $\frac{1}{2}$, *since this ratio is equal to* $\frac{1}{4}$ *divided by* $\frac{1}{2}$, and finally we may consider the last term as nothing, for the progression continually decreasing, this term will then be infinitely small, that is to say of an inappreciable smallness.

Thus, making use of the general formula $S = \dfrac{qx - a}{q - 1}$; we have $a = \frac{1}{2}$, $q = \frac{1}{2}$; and x considered

as nothing, which thus makes the term qx void; then therefore $S = \dfrac{-\frac{1}{2}}{\frac{1}{2}-1} = \dfrac{-\frac{1}{2}}{-\frac{1}{2}} = 1$, *the value of the sum required.*

LOGARITHMS.

Logarithms are numbers in Arithmetic progression, corresponding to a like number of terms in geometric progression. This is the most general idea that we can form of them.

But we shall speak here of that species of logarithms only, which are used to abridge the calculations in the operations of Arithmetic. *Tables* have been formed of them, from which they have received the name of *tabulary logarithms.*

We have taken for a geometric progression ∺ $(10)^0 : (10)^1 : (10)^2 : (10)^3$: &c. And as the exponents of the terms of this series, form an arithmetic progression, each exponent has been considered as the logarithm of the term to which it belongs.

The geometric progression being the same thing as ∺ 1 : 10 : 100 : 1000 : &c. we have then had immediately the logarithms of 1, 10, 100, &c. But it was wanted to find the logarithms of the intermediate numbers 2, 3, 4 - - - 15 - - - &c. for this effect, we have inserted so great a number of geometric proportional means between 1 and 10, between 10 and 100, between &c. that some of these terms were found to be exactly 2, 3, 4 &c. and having also inserted a like number of arithmetic proportional means between 0 and 1,

between 1 and 2, between &c. then we have had *for the logarithms of the numbers 2, 3, 4 &c.* the terms of the arithmetic progression, which were correspondent to them.

It is very evident, that the logarithm of every number comprised between 1 *and* 10, *is less than unity*; since the logarithm of 10 is exactly equal to unity; *that the logarithm of every number comprised between* 10 *and* 100, *will be unity, followed by a fractional part*; that &c. The fractional part of logarithms is always represented by decimals.

We call that part of the logarithm *characteristic*, which expresses the principal unities, and which is consequently on the left of the comma, which separates these unities from the decimal parts.

It will be observed, that the characteristic of a logarithm is equal to unity, taken as often less one, as the number to which this logarithm belongs contains figures: thus then, it is always easy to find the characteristic of the logarithm of any number. The characteristic of 7353 is 3.

The first advantage of logarithms is to change multiplication into addition, division into subtraction, involution into multiplication, and evolution into division. The simplicity of these operations, is owing to the essential property that *zero is the logarithm of unity*; for

First: *For Multiplication.* To multiply a number by another, is to add it to itself, as often as the multiplicator contains unity; the multiplicand, therefore, is contained in the product, as often as the multiplicator

contains unity. Thus if we designate the product by P, the multiplicand by M, and the multiplicator by m, we shall have this proportion $P : M :: m : 1$; but these four numbers being in geometric proportion, their logarithms will be in arithmetic proportion; that is to say (placing the letter L before each quantity to express its logarithm) we shall have $LP . LM : Lm . L1$, which gives $LP = LM + Lm - L1$; *but, as the logarithm of unity equals zero, it follows then, that the logarithm of the product, is equal to the sum of the logarithms of the two factors.*

Perhaps it is necessary to demonstrate what we have advanced, *that four terms being in geometric proportion, their logarithms are necessarily found in arithmetic proportion.*

We must consider that any one number, whether whole or fractional, can be supposed comprised in the geometric progression of a very great number of terms, which we have imagined established, to find the logarithms of all the *natural numbers*; for it can be conceived, that the progression rises insensibly in such a manner as to pass by such number as shall be wished; this being acknowledged, *we have seen, that, if in a geometric progression four such terms are taken, so that there shall be as many terms of the progression between the first and second, as between the third and the fourth, these four terms will be in proportion.*

From thence it follows, *vice-versa*, that if four terms, taken from a geometric progression, are in proportion, there will be as many terms of the progression comprised between the first and second, as between the third and the fourth. That being the case, it is very clear,

that if four numbers are in geometric proportion, their logarithms will be in arithmetic proportion, since, these holding the same rank in the arithmetic progression, as the others in the geometric progression, it follows, that there are as many terms of the arithmetic progression, between the log. of the two first terms, as between the log. of the two last, therefore these four logarithms form an arithmetic proportion.

Second: *For Division.* To divide a number by another, is to find how many times the first contains the second, and the quotient, which expresses this number of times, contains therefore unity as often as the dividend contains the divisor. Thus designating the quotient by Q, the dividend by D, and the divisor by d, we shall have this proportion $Q : 1 :: D : d$, and consequently $LQ . L1 : LD . Ld$.; this arithmetic proportion gives us $L . Q = L1 + LD - Ld$, *but as the log. of* 1 *equals zero, it follows, therefore, the log. of the quotient is equal to the log. of the dividend less the log. of the divisor.*

Third: *For Involution.* The square, or second power of a number, is the product of this number multiplied by itself; therefore the log. of this product or the log. of the second power of this number, is equal to twice the log. of this number. The cube, or the third power of a number, is the product of the square of this number by the number itself, therefore the log. of this product, or the log. of the third power of a number, is equal to thrice the log. of this number: from which it is evident, *that the log. of any power of a quantity is equal to the log. of the quantity multiplied by the exponent of the power to which this quantity is to be raised*; thus

$L a^2 = 2 L a$ - - - - - $L (a b)^5 = 5 L a b = 5 (L a + L b)$.

Fourth: *For Evolution.* It is evident that evolution being the contrary of involution, *we shall have the log. of any root of a number, by dividing the log. of this number by the index of the root.* We can be convinced of it in another manner, and make this case return into the preceding, by considering, that any root can be put into an exponential form; for to take, for example, the square root of a quantity, or to raise this quantity to the power $\frac{1}{2}$ is the same thing; now, to have the log. of a quantity raised to the power $\frac{1}{2}$, we must multiply the log. of this number by $\frac{1}{2}$, or which comes to the same, take the half of this logarithm. Generally $\sqrt[m]{a} = a^{\frac{1}{m}}$, therefore log. $\sqrt[m]{a} = \frac{1}{m} \times L a$.

RECAPITULATION. It follows, from the principles which we have just given, that $L a b = L a + L b$ - - - $L a b c = L a + L b + L c$ - - - $L (\frac{a}{b}) = L a - L b$ - - - $L (\frac{a b}{c}) = L a + L b - L c$ - - - $L (\frac{a}{c d}) = L a - L c - L d$ - - - $L a^m = m L a$ - - - $L \sqrt[m]{a} = \frac{L a}{m}$ - - - $L a^{\frac{m}{n}} = \frac{m L a}{n}$ - - - $L (a b)^{\frac{m}{n}} = \frac{m (L a + L b)}{n}$ - - - $L (\frac{a}{b})^{\frac{m}{n}} = \frac{m (L a - L b)}{n}$.

It is proper to forewarn beginners, that $L (a + b)$ cannot be put under another form, no more than $L (ab + cd)$; *the log. of a polynomial cannot be obtained but by adding together all the terms of the polynomial.*

This expression $\frac{La}{Lb}$ also merits attention; it is a logarithm to be divided by another log., and it must not be confounded with $L\left(\frac{a}{b}\right) = La - Lb$, which is the log. of the quotient of a divided by b.

THE USE OF THE TABLES.

The tables contain the logarithms of numbers only to a certain extent. There is no difficulty in finding the log. of a number contained in the tables; that log. is written on one side of the number; we find in the same way, a number whose logarithm is exactly contained in the tables.

But it may be necessary to find *the fractional number whose logarithm is comprised between two following log. of the tables*, or to find *this log. when we know the fractional number*. Often, also, *there is need of the log. of a number which exceeds those of the tables*, or finally, *to find this number when we know its logarithm*.

Four necessary questions to solve, to be able on every occasion to make use of the tables.

First Question. *To find the fractional number, whose log. is comprised between two consecutive log. in the tables.*

It is necessary to make a proportion, founded on this consideration, sufficiently exact, though not rigorous, *that the differences of logarithms which differ but little, may be supposed proportional to those of the numbers which they represent.*

Let there be, for example, such a log. as 2, 263944 which is comprised between the two log. of the tables 2, 264818 and 2, 262441. The numbers which correspond to these log. are 184 and 183. I have then this proportion 0,002367 *the difference between* 2,264818 and 2, 262451, *is to* 0,001493 *the difference between* 2,263944 and 2,262451, *as* 1 *the difference between* 184 and 183; that is to say, 0,002367 : 0,001493 :: 1 : *to the overplus of the number required above* 183, which overplus is found equal to $\frac{1493}{2367}$ = 0,63, *so that the number sought is* 183, 63 *nearly*, because the division has a remainder.

Second Question. *To find the log. of a fractional number comprised between two log. of the tables.*

It is obvious that we can here make use of the same proportion, and by supposing that the given number is 183 ,63, we shall say, then, if 1, *the difference between* 184 and 183, *is to* 0, 63, *the difference between the given number* and 183, *as* 0,002367 *the difference between the log. of* 183 and 184 *is to a fourth term*, which will be found to be 0,001493, which is the overplus of the log. of the given number above that of 183 ; by adding this excess to the log. of 183, we shall find the required log. which will be 2, 263944.

But there is a more expeditious method to be employed, whenever the decimal number is such, that by omitting the comma, the total number be found in the tables.——In the present case, the suppression of the comma would give us the number 18363, which is found in the tables, and the log. of which is 4, 263944 ——It is very plain now, that in order to have the log. of the required number, it is sufficient to take

away two units from the characteristic of that log. which gives 2, 263944 for the log. of 183 , 63.

Third Question. *To find the log. of a number which exceeds the tables.*

We must separate by the comma, on the right of this number, as many figures as shall be necessary, in order that the part on the left may be comprised in the tables. *Then nothing more is necessary than to find the log. of a fractional number, as in the preceding case*; afterwards the characteristic of this log. must be raised by as many unities as there were figures separated by the comma.

Let us suppose, (to make use of the example already given), that the tables extend no farther than ten thousand, and, that the number whose log. is desired, is 18363; this number exceeds the supposed tables, but I see that by separating two figures by the comma, I shall then have 183 , 63, which will be comprised in the tables, and having found, by the rules above, that the log. of this fractional number is, with a very little difference, 2 . 263944, then to have the log. of the required number, I shall augment its characteristic by two unities, and write 4, 263944, for, the characteristic of the log. of this number ought to have two unities more, than the characteristic of the log. of a number a hundred times smaller than it; and besides, we know also, that the characteristic is composed of as many unities less one, as the number contains figures.

Fourth Question. *A logarithm which exceeds the tables being given, to find the number to which this log. belongs.*

It is necessary to diminish the characteristic of the given log. so that this log. may be found in the tables, then taking the number which corresponds to this log. as many zeros must be written to its right, as unities have been suppressed in the characteristic.

However, it must be observed, that, if after having diminished the characteristic, the log. should not be found exactly comprised in the tables, the fractional number, which corresponds to it, must be sought for, as it has been shewn above; *this number necessarily containing decimal parts*, in order to have the required number, the comma must be removed towards the right, as many places, as there had been unities taken from the characteristic.

Let us suppose, then, as just now, that the tables of log. extend no farther than ten thousand, and that we have the log. 4, 263944, which could not be comprised in these tables; if I lower the characteristic by two unities, I shall have 2, 263944, which is found within the extent of the tables, and comprised between the log. of 184 and 183; I find by the given rules, that the fractional number which corresponds to this log. is, with a small difference, 183, 63, *then the required number is therefore* 18363.

These four operations, which we have just explained, ought to be practised, in order to be perfectly understood.

I add here some problems on logarithms, taken from a work of *Mr. Euler, a celebrated German geometrician.*

First Problem. *Suppose there should be a hundred thousand inhabitants in a province, and that the population augments there, every year a thirtieth part, it is asked, what will be the number of inhabitants in this province after a century?*

To facilitate the calculation, let us call n the present population. At the end of the first year this population will be $n + \frac{1}{30} \cdot n = n\,(1 + \frac{1}{30})$; at the end of the second year, it will be $n\,(1 + \frac{1}{30}) + \frac{1}{30} \cdot n\,(1 + \frac{1}{30})$; but let us remark, that in the two terms which compose this expression, we have $n\,(1 + \frac{1}{30})$, which is a common factor; then we shall be able, therefore, to put the whole quantity under this form $n\,(1 + \frac{1}{30}) \times (1 + \frac{1}{30}) = n\,(1 + \frac{1}{30})^2$; there will be found similarly, that at the end of the third year, the population will be $n\,(1 + \frac{1}{30})^3$, so that at the end of the century, it will be $n\,(1 + \frac{1}{30})^{100}$; putting in place of n its value, we shall have $100000\,(1 + \frac{1}{30})^{100} = x$ *the number sought for.*

But, if it was necessary to raise $1 + \frac{1}{30}$, or $\frac{31}{30}$, to the hundredth power by successive multiplications, it is very sensible, that the calculation would be of an extreme length; whereas, by making use of logarithms, we shall have immediately $L\,x = L\,100000 + L\,(\frac{31}{30})^{100} = L\,100000 + 100\,L\,\frac{31}{30}$; now $L \cdot \frac{31}{30} = L\,31 - L\,30 = 0,014241$, and then $100\,L\,\frac{31}{30} = 1,4241$; afterwards as $L\,100000 = 5$, we shall have, therefore, $L\,x = 5 + 1,4241 = 6,4241$, and seeking the number to which this log. belongs, *we shall find* $x = 2654800$, *which is the number of inhabitants that there would be in this province at the end of* 100 *years.*

Second Problem. *The earth having been repeopled after the deluge by the children of Noah and their three wives; we demand what must have been the increase each year, in order that the population of the earth might amount to one million at the end of two hundred years?*

This increase is supposed to have been always a same part of the population of each year. Let us call $\frac{1}{x}$ the increase of each year.

At the end of the first year, the population shall have been $6 + \frac{1}{x} \times 6 = 6\left(1 + \frac{1}{x}\right)$; by proceeding as in the preceding problem, it will be found, that at the end of the second year, it shall have been $6\left(1 + \frac{1}{x}\right)^2$, so that at the end of 200 years, it will be found $6\left(1 + \frac{1}{x}\right)^{200} = 1000000$, conformably to the enunciation of the question.

From this equation we get $1 + \frac{1}{x} = \left(\frac{1000\,000}{6}\right)^{\frac{1}{200}}$, and $L\left(1 + \frac{1}{x}\right) = \frac{1}{200} L\left(\frac{1000\,000}{6}\right)$; performing the calculations expressed in the second member, we find $L\left(1 + \frac{1}{x}\right) = 0{,}026109$. Now, seeking the fractional number, to which this log. answers, and pushing the operation to the sixth decimal figure, we shall have $1 + \frac{1}{x} = 1{,}061963$, from which we get $x + 1 = (1{,}061963)\,x$, and $x\,(0{,}061963) = 1$, from that $x = \frac{1}{0{,}061963}$, and finally, effecting the division, we find $x = 16$ nearly.

Therefore it was necessary, that the human race had increased every year $\frac{1}{16}$; which the robust health, and

the long days of our first parents, render sufficiently probable.

Third Problem. *In what proportion should a people increase every year, to be twice as numerous at the end of every century?*

Let n be the number of those that compose this people, let $\frac{1}{x}$ be the quantity required; we shall have for every secular epoch, the equation $n\left(1+\frac{1}{x}\right)^{100}=2n$, which gives $L\left(1+\frac{1}{x}\right)=L\frac{2}{100}=0,003010$; but the number to which this logarithm belongs, is $1,006955$, which gives $1+\frac{1}{x}=1,006955$, *from which we get* $x=144$ *nearly.*

Thus, (according to the remark of the author from whom we have taken these examples) *we ought to regard as very ridiculous the objections of those incredulous persons, who deny that the earth could have been peopled in so short a time by a single man.*

PRACTICAL

ARITHMETIC.

R

PRACTICAL ARITHMETIC.

NUMERATION.

EXAMPLES to learn how to write down a number read, and how to read a number written down.

427	Four hundred and twenty seven.
101	One hundred and one.
8073	Eight thousand and seventy three.
10090	Ten thousand and ninety.
704600	Seven hundred and four thousand, six hundred.
80007	Eighty thousand and seven.
111111	One hundred and eleven thousand, one hundred and eleven.
5002080	Five millions, two thousand, and eighty.
39000024	Thirty nine millions, and twenty four.
60000	Sixty thousand.

Decimal Numbers.

3,07	Three units, seven *hundredths.*
10,204	Ten units, two hundred and four *thousandths.*
45,0601	Forty five units, six hundred and one *ten-thousandths.*
2,000003	Two units, three *millionths.*
0,1024	One thousand, twenty four *ten-thousandths.*
0,00801	Eight hundred, and one *hundred-thousandths.*
0,010024	Ten thousand & twenty four *millionths.*
950,03	Ninety five thousand, three *hundredths.*
11,1	One hundred and eleven *tenths.*

Numbers are also expressed by letters, and are called *Roman* numbers, thus:

1	2	3	4	5	6	7	8	9	10	11
I,	II,	III,	IV,	V,	VI,	VII,	VIII,	IX,	X,	XI,
19	20	30	40	50	60	90	100	500		
XIX,	XX.	XXX.	XL,	L,	LX,	XC,	C,	D,		
1000	458	1049	1806							
M,	CDLVIII,	MXLIX,	MDCCCVI.							

It is to be remarked, that a letter of less value, standing before one of a greater, diminishes this; but when placed after, increases it.

ADDITION

OF INCOMPLEX NUMBERS.

Questions.

First. A merchant, on settling his accounts, finds he owes to A 95£. to B 3704, to C 820, to D 1001, and to E 954. I demand how much he owes in all? *Answer.* 6574£.

Second. Bought 8 casks of indigo; No. 1 weighs 210*lb*; No. 2, 196*lb*; No. 3, 4, 5, each 205*lb*; No. 6, 184*lb*; No, 7, 225*lb*; and No. 8, 174*lb*. How many pounds did the whole 8 casks contain? *Ans.* 1604*lb*.

Third. A Linen Draper bought 10 bales of linen cloth, containing as follows, *viz.* No. 1 and 2, each 367 *yards*; No. 3, 4, 5, each 407 *yards*; No. 6, 7, 8, each 288 *yards*; No. 9, 10, each 300 *yards*. How many *yards* did he buy in all? *Ans.* 3419 *yards*.

Fourth. A certain man being asked his age, answered: I have 7 sons, born at two years intervals the ones from the others; at the age of 21 years, I had my eldest son, and 21 is now the age of my youngest. It is required to tell his age? *Ans.* 54 *years*.

Fifth. If from the *Creation* to the *flood* have elapsed 1650 years; from the *flood* to the *Vocation of Abraham*, 427; from the *vocation of Abraham*, to the *founding of the Temple*, 1010; from *that* to the *foundation of Rome*, 266; thence to the *birth of Christ*, 752. How long is it since the Creation? *Ans.* (*in the year* 1805.) 5910 *years*.

SUBTRACTION

OF INCOMPLEX NUMBERS.

Questions.

First. What number is that, which being added to 977, the sum will be 2081? *Ans.* 1104. 1104

Second. What number must I subtract from 3002, that the remainder may be 1104? *Ans.* 1898.

Third. A vintner bought 20 *Pipes* of Brandy, containing 2459 *Gallons*, and sells 14 *pipes*, containing 1680 *gallons*; how many *pipes* and *gallons* has he unsold? *Ans.* 6 *pipes, containing* 779 *gallons.*

Fourth. I bought 2000 *yards* of linen for 466£, and sold 1476 *yards* for 369£; How many *yards* have I left, and what do I want to make up the first cost? *Ans.* 524 *yards, and want* 97£.

Fifth. If I was born in the year 1756, how many years old am I? *Ans.* (*in the year* 1805) 49 *years.*

Sixth. Four notable discoveries were made in 190 years time, *viz.* 1st, the invention of the *Compass*; 2d, *Gunpowder*; 3d, *Printing*, 4th, *America*; the last was discovered *anno* 1492; the third 52 years before; the second 42 years after the first; the question is, in what year did each discovery happen? *Ans. Compass in* 1302, *Gunpowder* 1344, *Printing* 1440.

MULTIPLICATION
OF INCOMPLEX NUMBERS.

THE TABLE.

TWICE	3 TIMES	4 TIMES	5 TIMES	6 TIMES	7 TIMES
2 *make* 4	2 — 6	2 — 8	2 — 10	2 — 12	2 — 14
3 — 6	3 — 9	3 — 12	3 — 15	3 — 18	3 — 21
4 — 8	4 — 12	4 — 16	4 — 20	4 — 24	4 — 28
5 — 10	5 — 15	5 — 20	5 — 25	5 — 30	5 — 35
6 — 12	6 — 18	6 — 24	6 — 30	6 — 36	6 — 42
7 — 14	7 — 21	7 — 28	7 — 35	7 — 42	7 — 49
8 — 16	8 — 24	8 — 32	8 — 40	8 — 48	8 — 56
9 — 18	9 — 27	9 — 36	9 — 45	9 — 54	9 — 63
10 — 20	10 — 30	10 — 40	10 — 50	10 — 60	10 — 70
11 — 22	11 — 33	11 — 44	11 — 55	11 — 66	11 — 77
12 — 24	12 — 36	12 — 48	12 — 60	12 — 72	12 — 84

8 TIMES	9 TIMES	10 TIMES	11 TIMES	12 TIMES
2 — 16	2 — 18	2 — 20	2 — 22	2 — 24
3 — 24	3 — 27	3 — 30	3 — 33	3 — 36
4 — 32	4 — 36	4 — 40	4 — 44	4 — 48
5 — 40	5 — 45	5 — 50	5 — 55	5 — 60
6 — 48	6 — 54	6 — 60	6 — 66	6 — 72
7 — 56	7 — 63	7 — 70	7 — 77	7 — 84
8 — 64	8 — 72	8 — 80	8 — 88	8 — 96
9 — 72	9 — 81	9 — 90	9 — 99	9 — 108
10 — 80	10 — 90	10 — 100	10 — 110	10 — 120
11 — 88	11 — 99	11 — 110	11 — 121	11 — 132
12 — 96	12 — 108	12 — 120	12 — 132	12 — 144

Examples.

1st.	*Multiply*	7643827	*by*	23	Product.	175808021
2nd.		8142630	*by*	75		610697250
3d.		56071592	*by*	123		6896805816
4th.		275827	*by*	19725		5440687575
5th.		327586	*by*	603		197534358
6th.		7864371	*by*	20604		162037500084
7th.		5070984	*by*	4050607		20540563287288
8th.		6247386495	*by*	27356		170903504957220

Decimal Numbers.

1st	*Multiply*	97,105	*by*	8,6	Product	834,630
2nd.		380,4	*by*	9,53		3625,212
3d.		57	*by*	0,508		28,956
4th.		0,14	*by*	0,009		0,00126

Proof.

We have given in the first part different ways for making the proof of multiplication, but there is another very short, which I will give here, though not infallible,* *it is called the proof of multiplication by the cross, or the proof of multiplication by 9.*

Make a cross thus X, then add all the figures in the multiplicand together, as in Addition, and cast away the nines, as often as they arise, and bear the remainder to the next figure; when you come to the end of the line, note what remains after the nines are cast away, and set such remainder on the left side of the cross.

Then do the same by the multiplier, and note what remains there also, setting that on the right of the

* The proof may be found right, and the fum be wrong, when, in the product, a cypher is put for a nine, or when the places of the figures are changed.

cross; multiply these two figures together, and cast the nines out of the product, setting the remainder on the top of the cross.

Lastly, cast away the nines out of the product, and if the remainder be like the figure on the top of the cross, you may hope that the work is right.

I will give one example, to make the foregoing directions the more intelligible.

	3			56071592	*multiplicand*
8	X	6		123	*multiplier*
	3			6896805816	*Product.*

Here, after having made the cross, I begin at the multiplicand, saying, 5 and 6 is 11, I cast away 9, and there remains 2; now 2 and 7 is 9, and there rests nothing; then 1 and 5 is 6 (I skip the 9) and 2 is 8, which I set on the left hand of the cross, as it is seen.

Going to the multiplier, I say, 1 and 2 is 3 and 3 is 6, which I place on the right of the cross. Then I multiply 8 by 6, and of the product 48, casting away the nines there remains 3, which I write on the top of the cross.

After the same manner, I cast away the nines out of the product, and at the last there remains 3 likewise, and so the proof is done.

Contractions.

1st. *When the multiplier is* 10 or 100, or 1000, &c.

The rule has been given in the 1st part (page 17.)

2nd. *When there are cyphers at the right hand, in both the multiplicand and multiplier.*

These two cases have been explained in the first part, and are not in need of examples.

3d. *When the multiplier is* 11 *or* 12.

Multiply at once each figure of the multiplicand by the two figures of the multiplier, so that the product may be in one line only.

EXAMPLES.

7469	8793
11	12
82159	105516

4th. *When the multiplier is* 5.

Annex a cypher to the multiplicand, then halve it, because 5 is the $\frac{1}{2}$ of 10.

When the multiplier exceeds 12, *and is less than* 20.

Write under the multiplicand, its product by the unit figure of the multiplier, and set the first figure of this product one place forward to the right hand.

EXAMPLES.

Multiply	78395	*by* 13,	*and*	6754	*by* 18
	235185			54032	
	1019135	- *Products* -		121572	

Or this way, that the product may be in one line only.

Multiply	78395	*Multiply*	6754
by	13	*by*	18
	1019135 - *Products* -		121572

Multiply by the unit figure, and add to the product of each figure, that which is next on the right hand.

QUESTIONS.

First. The sum of two numbers is 360; the less is 114. What is the product of these two numbers? *Ans.* 28044.

Second. There are two numbers; the greater of them is 73 times 109, and their difference 17 times 28. We demand the product of these two numbers? *Ans.* 59526317.

Third. I bought 10 bales of cloth; in each bale 59 pieces; in each piece 25 yards; and the price of each yard is 300 *cents*. What sum must I pay? *Ans.* 4425000 *cents*.

Fourth. A Merchant bought 11 bales of cloth; in each bale 1475 yards; for each yard he paid 120 *cents*; this merchant sold 13928 yards at 135 cents each. How much do his receipts fall short of his expenditure? *Ans.* 66720 *cents*.

Fifth. A Vintner bought 5 hogsheads of brandy, for the sum of 157 *dollars*; he sold the quart 18 *cents*; it is known, that one hogshead contains 252 *quarts*, and that one *dollar* is worth 100 cents. What was the gain of the Vintner? *Ans.* 6980 *cents*.

DIVISION

OF INCOMPLEX NUMBERS.

Examples.

1st.	*Divide*	78901	*by*	32	*Quot.*	2465 - 21	*Rem.*
2nd.	- - -	16790	*by*	46	- - - -	365 - 0	
3d.	- - -	60329	*by*	754	- - - -	80 - 9	
4th.	- - -	345678	*by*	942	- - - -	366 - 906	
5th.	- - -	589432	*by*	58	- - -	10162 - 36	
6th.	- -	7450946	*by*	742	- - -	10041 - 524	
7th.	-	345678901	*by*	2345	- -	147411 - 106	
8th.	-	197534358	*by*	327583	- -	603 - 1809	

Decimal Numbers.

1st.	*Divide*	387,39	*by*	5,57	*Quotient*	69 - 306	*Rem.*
2nd.	- - -	890,4	*by*	6,27	- - -	142 - 6	
3d.	- - - -	59,1	*by*	0,036	- -	1641 - 24	
4th.	- - -	742	*by*	0,07	- -	10600 - 0	

Divide
- 624 *by* 37 *and get the quotient within a tenth?* Ans. 16,8.
- 95,35 *by* 48 *and get the quotient within a hundredth?* Ans. 1,98.
- 0,8 *by* 0,009 *and get the quotient within a hundredth?* Ans. 88,88.

Contractions.

When the Divisor is terminated by cyphers.

The rule has been given in the first part (page 23.)

EXAMPLES.

1st.	*Divide*	425890	*by*	4500	*Quotient*	$94 + \frac{289}{450}$
2nd.	- - -	78400	*by*	340	- - - - -	$207 + \frac{2}{34}$
3d.	- - -	564000	*by*	9000	- - - - -	$62 + \frac{6}{9}$
4th.	- - -	862379	*by*	85000	- - - - -	$10 + \frac{12379}{85000}$

When the Divisor does not exceed 12.

EXAMPLES.

1st. *Divide* 4474 *by* 7 *Quotient* 639 + $\frac{1}{7}$
2nd. - - - 97961 *by* 9 - - - 10884 + $\frac{5}{9}$
3d. - - - 53242 *by* 12 - - - 4436 + $\frac{10}{12}$

EXPLICATION. *In the first Example*, we have said, the seventh of 44 is 6, but it remains 2; that, with the following figure, makes 27; now the seventh of 27 is 3 for 21, it remains 6; that, with the following figure, makes 64; and the seventh of 64 is 9 with the remainder 1.

We have taken, by the same way, the ninth of the Dividend in the second example, and the twelfth in the third.

When the Divisor is an exact product of two numbers, which do not exceed 12.

Divide first by one of the two numbers, and that quotient by the other number; then multiply the first divisor into the last remainder, if any, and to that product add the first remainder *for the true one.*

Let 1212287 be given to be divided by 24=6×4.

First I divide by 6 according to the preceding rule, and the quotient is 202047, with a rest 5; now I divide 202047 by 4, which gives me 50511 for the quotient desired, and 3 for remainder; then I say, 6 times 3 is 18, to which adding 5, the first remainder, I have 23 *for the true one.*

Examples.

1st.	*Divide* 73913	*by* 32	*Quotient*	2309	- 25	*Rem.*
2nd	- - 238595	*by* 49	- - - -	4869	- 14	
3d.	- - - 584326	*by* 81	- - - -	7213	- 73	
4th.	- - - 95287	*by* 132	- - - -	721	- 115	

Proof.

We have given in the first part, the rule for the proof of Division, but it can be done also by the cross; for the dividend being equal to the product of the divisor by the quotient, more the remainder, we must, after having followed the rule for multiplication, take the nines out of the remainder, add the rest to that obtained by the aforesaid rule, and take the nines again, if necessary.

Questions.

First. The product of two numbers is 183451848, the multiplicand 24. What is the multiplier? *Ans*. 7643827.

Second. The product of two numbers is 86812764, the multiplier is 943617. What is the multiplicand? *Ans*. 92.

Third. I bought 112 dozen of knives, and paid 13440 cents. How many cents was each knife? *Ans*. 10.

Fourth. Several boys went to gather nuts, and collected 4560, which when distributed among them, each had 285. How many boys were in company? *Ans*. 16.

Fifth. I paid 1401 dollars for 467 yards. How much each yard? *Ans.* 3 dollars.

Sixth. A merchant has three clerks; to the firs he gives 780 dollars *per year*; to the second 35 *per month*; to the third 5 *per week*. He wishes to know, how much he has to pay per day? *Ans.* 4 *dollars.*

FRACTIONS.

Examples.

To Reduce fractions to a common denominator by the general method, and add them.

1st. $\frac{2}{3} + \frac{4}{5} + \frac{6}{7} = \frac{244}{105}$

2nd. $\frac{5}{7} + \frac{1}{2} + \frac{8}{9} + \frac{3}{5} = \frac{1703}{630}$

3d. $\frac{2}{5} + \frac{1}{6} + \frac{3}{4} + \frac{4}{7} = \frac{1586}{840}$

4th. $\frac{3}{11} + \frac{5}{9} + \frac{7}{10} = \frac{1513}{990}$

A Whole Number with Fractions.

5th. $2 + \frac{5}{6} + \frac{7}{8} + \frac{4}{7} = \frac{1438}{336}$

6th. $5 + \frac{11}{12} + \frac{3}{7} + \frac{1}{5} = \frac{2749}{420}$

Contractions.

1st Case. $\begin{cases} \frac{3}{8} + \frac{5}{10} + \frac{1}{4} + \frac{7}{40} = \frac{52}{40} \\ \frac{2}{9} + \frac{7}{12} + \frac{5}{36} + \frac{1}{6} = \frac{40}{36} \end{cases}$

2nd Case. $\begin{cases} \frac{4}{9} + \frac{3}{8} + \frac{1}{2} + \frac{5}{24} = \frac{110}{72} \\ \frac{2}{5} + \frac{5}{6} + \frac{9}{10} + \frac{1}{15} = \frac{66}{30} \end{cases}$

3d Case. $\begin{cases} \frac{4}{15} + \frac{7}{9} + \frac{5}{12} + \frac{1}{27} = \frac{809}{540} \\ \frac{2}{21} + \frac{3}{7} + \frac{1}{35} + \frac{5}{6} = \frac{291}{210} \end{cases}$

Proof.

Add the sum of the complementary* fractions to that of the given one, the work will be right, if these two sums together, are equal to as many units as there are fractions.

Application.

Complementary fractions of the 1st foregoing examples. $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{71}{105}$; now adding this sum with that of $\frac{244}{105}$, I have $\frac{315}{105} = 3$.

2nd. Example. Complementary fractions $\frac{2}{7} + \frac{1}{2} + \frac{1}{9} + \frac{2}{5} = \frac{817}{630}$, and then $\frac{817}{630} + \frac{1703}{630} = \frac{2520}{630} = 4$.

SUBTRACTION.

Examples.

1st. $\frac{8}{9} - \frac{3}{7} = \frac{29}{63}$

2nd. $\frac{5}{6} - \frac{1}{2} - \frac{1}{7} = \frac{16}{84}$

3d. $\frac{2}{3} - \frac{1}{4} - \frac{5}{6} + \frac{7}{12} = \frac{9}{24}$

A Mixed Number from a Mixed Number.

First. *When the fraction to be subtracted is the less*, subtract the less fraction from the greater fraction, and the less *whole* from the greater.

from	$2\frac{3}{4}$	$5\frac{1}{2}$	$7\frac{2}{3}$	$26\frac{1}{4}$
take	$1\frac{2}{3}$	$3\frac{1}{4}$	$6\frac{2}{5}$	$21\frac{1}{6}$
differences	$1\frac{1}{12}$	$2\frac{1}{4}$	$1\frac{4}{15}$	$5\frac{1}{12}$

* We call that a complementary fraction, which is wanted to a fraction to be equal to the unit.

Second. *When the Fraction to be subtracted is the greater.* Borrow one unit from the number annexed to the smallest fraction, then adding that unit to this fraction, you make the Subtraction.

$\left.\begin{array}{l}\textit{from } 3\frac{2}{5}\\ \textit{take } 1\frac{3}{4}\end{array}\right\}$ or $\left\{\begin{array}{l}\textit{from } 2\frac{7}{5}\\ \textit{take } 1\frac{3}{4}\end{array}\right\}$ *gives* $1\frac{13}{20}$ *for the difference.*

MULTIPLICATION.

Be a Mixed Number to multiply by a Fraction; or, a Mixed Number by another Mixed Number.

The shortest way in the two cases, is to add together the two parts of each mixed number, and then the question is reduced to multiply one fraction by another.

EXAMPLES.

1st. $\left.\begin{array}{l}\textit{Multiply } 7\frac{3}{4}\\ \textit{by } \frac{2}{5}\end{array}\right\}$ *or,* $\frac{31}{4}$ *by* $\frac{2}{5}$, *which gives for the product* $\frac{62}{20} = 3\frac{2}{20}$

2nd. $\left.\begin{array}{l}\textit{Multiply } 9\frac{4}{5}\\ \textit{by } 6\frac{2}{7}\end{array}\right\}$ *or,* $\frac{49}{5}$ *by* $\frac{44}{7}$, *which gives* $\frac{2156}{35} = 61\frac{21}{35}$.

DIVISION.

The same observation for Division as for Multiplication, with regard to the mixed numbers.

EXAMPLES.

1st. $\left.\begin{array}{l}\textit{Divide } 5\frac{1}{6}\\ \textit{by } \frac{2}{7}\end{array}\right\}$ *or,* $\frac{31}{6}$ *by* $\frac{2}{7}$, *which gives for result* $\frac{217}{12} = 18\frac{1}{12}$.

2nd. $\left.\begin{array}{l}\textit{Divide } 6\frac{5}{8}\\ \textit{by } 11\frac{4}{9}\end{array}\right\}$ *or,* $\frac{53}{8}$ *by* $\frac{103}{9}$, *which gives for result* $\frac{477}{824}$.

FRACTIONS OF FRACTIONS.

QUESTIONS.

First. A Merchant buys the $\frac{4}{9}$ of the cargo of a vessel; then he makes over to one of his friends the $\frac{2}{5}$ of his purchase. What proportion has each of them of the total cargo of the Vessel? *Ans.* $\frac{12}{45}$ & $\frac{8}{45}$

Second. Nine persons are to divide equally among themselves the sum of 8283£. One of these persons dying, leaves a widow and four children; he bequeaths to his widow the third part of his claim, and his children must divide the rest in equal shares. *Query*: How much will the widow and each of the children receive. *Ans.* The widow, $306\frac{7}{9}$, and each of the children, $153\frac{14}{36}$.

Third. Peter and Paul are in partnership, but Peter, who owns the $\frac{2}{3}$ of the capital, dies, leaving a wife and 3 children; he wills to his widow the $\frac{2}{7}$ of his estate, to the eldest of his children *one third and a half of the third* of what remains, and the other two children must divide equally the remainder. It is supposed that the whole capital in partnership is 54327£. What are the shares of the widow and of each of the children? *Ans.* The widow 10348, the eldest 12935, each of the others $6467\frac{1}{2}$.

REDUCTION OF FRACTIONS.

(By way of Trial.)

EXAMPLES.

1st. $\frac{12600}{17640} = \frac{5}{7}$	5th. $\frac{617400}{1852200} = \frac{1}{3}$
2nd. $\frac{4725}{7875} = \frac{3}{5}$	6th. $\frac{240786}{277830} = \frac{13}{15}$.
3d. $\frac{66150}{110250} = \frac{3}{5}$	7th. $\frac{354375}{401625} = \frac{15}{17}$.
4th. $\frac{44100}{61740} = \frac{5}{7}$	

By the way of the Greatest Common Divisor.

In order to exercise the scholars in this kind of Reduction, Examples may be given to them in this manner. Make them multiply two fractions by each other, then to make the proof of this multiplication, let them divide the product by one of the factors, the quotient of this division, after the reductions made, will be the other factor.

APPLICATION.

$\frac{11}{15} \times \frac{7}{8} = \frac{77}{120}$; then $\frac{77}{120}$ *div.* $\frac{11}{15} = \frac{1155}{1320}$; this last fraction being reduced to its lowest terms by the way of the greatest common divisor, gives $\frac{7}{8}$, as it must be.

To Reduce a Vulgar Fraction into Decimals.

Write 1, 2, 3, &c. cyphers at the right of the numerator of the fraction ; then divide this number, thus increased, by the denominator ; the quotient will contain as many decimals, as cyphers you will have written at the right of the numerator.

APPLICATION.

It is required to express the value of the fraction $\frac{5}{7}$ within one thousandth part.

Since the quotient must contain thousandths, you must write three cyphers at the right of the numerator which gives 5000 to be divided by 7, the quotient of which is 714, and consequently the value of the fraction $\frac{5}{7}$ within a thousandth, will be 0,714.

In like manner a Decimal fraction can be placed under the form of a vulgar fraction, for it is evident that 0,714, for example, can be written thus, $\frac{714}{1000}$.

Of COMPLEX, or COMPOUND NUMBERS.

DENOMINATIONS.

Money.

4 *farthings*, (marked *qr.*) make 1 *penny*, marked *d.*
12 *pence* - - - - - - - 1 *shilling* - - *s.*
20 *shillings* - - - - - - - 1 *pound* - - *£.*

Table.

Farthings.	*Pence.*	*Shillings.*	*Pound.*
4	1		
48	12	1	
960	240	20	1

FEDERAL MONEY.

10 mills (*m*) make 1 cent - - - *c.*
10 cents - - 1 dime - - *d.*
10 dimes - - 1 dollar - - *D.* or *$.*
10 dollars - - 1 eagle - - *E.*

TROY WEIGHT.

By this weight, jewels, gold, silver and liquors are weighed.

24 *grains* (grs.) make 1 *pennyweight* - *dwt.*
20 *pennyweights* - - 1 *ounce* - - - *oz.*
12 *ounces* - - - 1 *pound* - - - *lb.*

TABLE.

Grains.	P. weights.	Ounces.	Pound.
24	1		
480	20	1	
5760	240	12	1

AVOIRDUPOIS WEIGHT.

By this weight are weighed goods of all kinds, *the above excepted.*

16 *drams* (*dr.*) make 1 *ounce* - - - - - *oz.*
16 *ounces* - - - 1 *pound* - - - - - *lb.*
14 *pounds* - - - 1 *stone*
28 *pounds* - - - 1 *quarter of an* { *hundred weight* } *qr.*
4 *quarters* - - - 1 *hundred-weight* - - *Cwt.*
20 *hundred-weight* - 1 *ton* - - - - - *T.*

TABLE.

Drachms	*Ounces.*	*Pounds.*	*Stones.*	*Quarters.*	*Cwt.*	*Ton.*
16	1					
256	16	1				
3584	224	14	1			
7168	443	28	2	1		
28672	1792	112	8	4	1	
573440	35840	2240	160	80	20	1

APOTHECARIES WEIGHT.

By this weight Apotheearies compound their medicines; but buy and sell by Avoirdupois-weight.

20 *Grains* (*gr.*)	make	1 *Scruple*,	℈
3 *Scruples*	- - -	1 *Drachm*,	ʒ
8 *Drachms*	- - -	1 *Ounce*,	℥
12 *Ounces*	- - -	1 *Pound*,	℔

LONG MEASURE.

Long Measure is used for lengths or distances.

3 *barley-corns* (*b. c.*) make 1 *inch* - - - - *in.*
12 *inches* - - - - - - 1 *foot* - - - - *ft.*
3 *feet* - - - - - - 1 *yard* - - - - *yd.*
2 *yards* - - - - - - 1 *fathom* - - - *fa.*
$5\frac{1}{2}$ *yards* - - - - - 1 *rod, pole,* or *perch* *P.*
40 *poles* - - - - - - 1 *furlong* - - - *fur.*
8 *furlongs* - - - - 1 *mile* - - - - *M.*
3 *miles* - - - - - - 1 *league* - - - *L.*
60 *geographic* or / $69\frac{1}{2}$ *statute* } *miles* - 1 *degree* - - *deg.*
360 *degrees* - - - *a great circle of the earth.*

TABLE.

Inches.	*Feet.*	*Yards*	*Fathoms.*	*Poles* or *Perch.*	*Furlongs.*	*Mile.*
12	1					
36	3	1				
72	6	2	1			
198	$16\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{3}{4}$	1		
7920	660	220	110	40	1	
63360	5280	1760	880	320	8	1

SQUARE MEASURE,

Is used in finding the contents of surfaces when we measure both the length and breadth.

144 *square inches* make	1 *square foot*	- - - -	*Ft.*
9 *square feet* - -	1 *square yard*	- - -	*Yd.*
4 *square yards* - -	1 *square fathom*	- -	*F. m.*
$30\frac{1}{4}$ *square yards* -	1 *perch*	- - - - - -	*P.*
40 *perches* - - -	1 *rood*	- - - - - -	*R.*
4 *roods* - - -	1 *acre*	- - - - - -	*A.*

TABLE.

SQUARE

Feet.	*Yards.*	*Fathoms.*	*Perch.*	*Roods.*	*Acre.*
9	1				
36	4	1			
$272\frac{1}{4}$	$30\frac{1}{4}$	$7\frac{9}{16}$	1		
10890	1210	$302\frac{1}{2}$	40	1	
43560	4840	1210	160	4	1

CLOTH MEASURE.

4 *nails* (*na*) make	1 *quarter of a yard*	- - -	*qr.*
4 *quarters* - -	1 *yard*	- - - - - - -	*yd.*
3 *quarters* - -	1 *ell Flemish*	- - - - -	*E. Fl.*
5 *quarters* - -	1 *ell English* or *French.*		*e. E. e. Fr.*
$2\frac{1}{4}$ *quarters* or 10 *nails*	1 *ell Hamburgh*	- - -	*E. H.*

LIQUID MEASURE.

2 *Pints* (*pt.*) make 1 *quart* - - - - *qt.*
4 *quarts* - - - - 1 *gallon* - - - - *gal.*
42 *gallons* - - - - 1 *tierce* - - - - *ti.*
63 *gallons* - - - - 1 *hogshead* - - - *hhd.*
2 *hogsheads* - - - 1 *pipe* or *butt* - *pi.* or *bt.*
2 *pipes* or 4 *hogsheads* 1 *ton* - - - - - *T.*

TABLE.

Pints	*Quarts.*	*Gallons.*	*Tierces.*	*Hogsheads.*	*Pipes.*	*Tun.*
2	1					
8	4	1				
336	168	42	1			
504	252	63	$1\frac{1}{2}$	1		
1008	504	126	3	2	1	
2016	1008	252	6	4	2	1

DRY MEASURE.

This measure is used for grain, fruit, salt, &c.

2 *pints* (*pt.*) make 1 *quart* - - - - *qt.*
8 *quarts* - - - - 1 *peck* - - - - *P.*
4 *pecks* - - - - 1 *bushel* - - - - *bu.*

TABLE.

Pints.	*Quarts.*	*Pecks.*	*Bushel.*
2	1		
16	8	1	
64	32	4	1

TIME.

60 *seconds* (*sec.*) make 1 *minute* - - *Min.*
60 *minutes* - - - - - 1 *hour* - - - - *H.*
24 *hours* - - - - - 1 *day* - - - - *D.*
7 *days* - - - - - 1 *week* - - - *W.*
365 *days* - - - - - 1 *year* - - - - *Y.*

TABLE.

Minutes	*Hours.*	*Days.*	*Weeks.*	*Year.*
60	1			
1440	24	1		
10080	168	7	1	
525600	8760	365	$52\frac{1}{7}$	1

NOTE. *Every fourth year, called leap-year, is of* 366 *days.*

The year is also divided into 12 calendar months, as follows:

The fourth, eleventh, ninth, and sixth,
Have thirty days to each affix'd;
And every other thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
'Till leap-year gives it twenty-nine.

MOTION, OR CIRCLE-MEASURE.

This is used by Astronomers, Navigators, &c.

60 *seconds*	(") make	1 *minute*	- - - -	'
60 *minutes*	- - - -	1 *degree*	- - - -	°
30 *degrees*	- - - -	1 *sign*	- - - -	*sig.*
12 *signs*, or 360 *degrees*		1 *circle*		

ADDITION.

EXAMPLES OF MONEY.

£. s. d.	£. s. d.
345 - 15 - 8	17 - 12 - 3½
24 - 7 - 10	1546 - 6 - 10¾
3343 - 13 - 11	820 - 19 - 7¼
0 - 18 - 7	3729 - 17 - 11½
924 - 5 - 10	703 - 3 - 11¼
73 - 9 - 6	527 - 5 - 4¾
5725 - 0 - 9	9708 - 0 - 10¼
10937 - 12 - 1	17053 - 6 - 11¼

Remarks.

1st. The addition of fractions is made, on similar occasions, in a very easy manner, by mentally placing all the fractions to the same Denominator ; thus, for instance, in the above example, each $\frac{1}{2}$ has been counted as $\frac{2}{4}$; and then adding all the numerators, we have had 13 to divide by the common Denominator 4, which has given 3 for quotient, which has been carried to the column of pence, and $\frac{1}{4}$ for remainder, which has been written after the pence in its fractional form.

2d. In order to facilitate the addition of pence, it is proper to sum up first the figures of units, then you add the tens, to form the total sum of the pence.

3d. To make the addition of shillings, sum up first the figures of units, then carrying the tens of this sum, count them as units, and add them up to the tens of the shillings ; then take the half of them, which are so many pounds.

TROY WEIGHT.

lb.	*oz.*	*dwt.*	*gr.*	*lb.*	*oz.*	*dwt.*	*gr.*
879	11	14	23	53	10	19	4
53	10	16	21	199	7	11	23
564	3	7	12	86	11	15	18
432	9	18	3	27	9	3	4
607	2	15	20	6	10	13	8
9	10	12	18	341	5	14	21
2548	1	6	1	716	7	18	6

Remarks.

1st. To sum up the grains, it is very convenient to mark a dot upon your nail for each sum equal to

24; so that you will have as many units to carry to the column of *dwt.* as dots you will have marked.

2nd. The addition of *dwt.* is made as that of shillings, and the addition of ounces like that of pence.

AVOIRDUPOIS WEIGHT.

T.	C.	*qrs.*	*lb.*	C.	*qrs.*	*lb.*	*oz.*
7	12	3	21	54	2	13	15
10	15	2	14	17	1	25	4
3	7	1	9	3	0	27	12
2	18	0	20	19	3	4	10
5	2	1	15	78	1	21	3
0	9	3	7	10	2	6	9
30 T.	6 C.	1 *qr.*	2 *lb.*	184 C.	0 *qrs.*	15 *lb.*	5 *oz.*

Remarks.

1st. An easy method for the addition of pounds, is to add up first the figures of units, and for every sum of 28 mark a dot on your nail; the addition of units being finished, you carry the remainder to the tens, which are each of them counted as ten simple units, and you continue to form, successively, sums equal to 28, which you mark on your nail as they come, and finally you write what remains, at the foot of the column.

2nd. For the hundred weights, you must work in the 1st. example, according to the method given for adding up shillings.

3d. Make the additions of ounces, according to the direction just given for that of pounds.

LONG MEASURE.

yd.	ft.	in.	fm.	ft.	in.
357	2	9	734	5	9
983	1	11	502	4	7
58	0	10	617	2	10
289	1	7	99	3	11
704	2	3	843	5	3
91	2	11	90	1	8
2486 *yd.*	0 *ft.*	3 *in.*	2889 *fm.*	0 *ft.*	0 *in.*

LIQUID MEASURE.

T.	hhd.	gal.	gal.	qt.	pt.
3	2	43	327	3	0
2	1	24	729	2	1
4	3	60	215	1	1
1	1	49	508	3	1
7	2	27	53	1	1
0	3	58	137	3	1
21 T.	0 *hhd.*	9 *gal.*	1972 *gal.*	3 *qt.*	1 *pt.*

SUBTRACTION.

Examples.

	£.	s.	d.			£.	s.	d.
1st. from	$34\frac{1}{3}$			or	from	34	6	8
subtract	21	12	7		subtract	21	12	7

After this preparation, no difficulty remains.

	£.	s.	d.			£.	s.	d.
2nd. from	$15\frac{3}{7}$			or	from	15	8	$6\frac{6}{7}$
subtract	12	9	$4\frac{2}{5}$		subtract	12	9	$4\frac{2}{5}$

	£.	s.	d.			£.	s.	d.
3d. from	7	9	$3\frac{1}{5}$	or	from	7	9	$3\frac{1}{5}$
subtract	$5\frac{2}{7}$				subtract	5	5	$8\frac{4}{7}$

Let it suffice to have indicated these few cases, which might have puzzled the beginners at first sight.

MULTIPLICATION.

EXAMPLES.

	£. s. d.		£. s. d.
1st. *multiply*	135 - 13 - 5	*product*	26727 - 3 - 1
by	197		
	£. s. d.		£. s. d.
2nd. *mult.*	79 - 0 - 10	*product*	26241 - 16 - 8
by	332		
	£. s. d.		£. s. d.
3d. *mult.*	354 - 16 - 10	*product*	73452 - 4 - 6
by	207		
	fm. ft. in.		*fm. ft. in.*
4th. *mult.*	37 - 0 - 5	*product*	7599 - 1 - 5
by	205		
	fm. ft. in.		*fm. ft. in.*
5th. *mult.*	119 - 4 - 11	*product*	69016 - 0 - 0
by	576		
	T. C. *qrs.*		T. C. *qrs.*
6th. *mult.*	28 - 12 - 2	*product*	12337 - 7 - 2
by	431		
	T. C. *qrs.*		T. C. *qrs.*
7th. *mult.*	46 - 15 - $3\frac{4}{5}$ *	*product*	7627 - 19 - $3\frac{2}{5}$
by	163		
	C. *qrs. lb.*		C. *qrs. lb.*
8th. *mult.*	236 - 3 - 19	*product*	42408 - 2 - 13
by	179		
	C. *qrs. lb.*		C. *qrs. lb.*
9th. *mult.*	409 - 2 - 22	*product*	269477 - 3 - $8\frac{1}{2}$
by	$657\frac{3}{4}$		

* With respect to the fraction, you must resolve it into $\frac{1}{5}$ and $\frac{3}{5}$, so that having the product for 1 *qr.* you take the fifth of this product, then three times that; this proceſs is the plainest in all similar cases.

QUESTIONS.

First. If 1 fathom cost 34£. 19s. 7d. what cost 245*fm.* 4*ft.* 10*in*? *Ans.* 8598£. 1*s.* $5\frac{23}{36}$*d.*

Second. If for 1 pound, I get 4*fm.* 0 *ft.* 8*in.* how many for 178£. 13*s.* 5*d*? *Ans.* 734*fm.* 3*ft.* $2\frac{17}{30}$*in.*

Third. How much will cost 316 C. 3*qrs.* 11*lb.* if 1 C. cost 7£. 9*s.* 8*d.*? *Ans.* 2371£. 1*s.* $7\frac{11}{26}$*d.*

Fourth. If for 1£. I get 2*lb.* 9*oz.* 13*dwt.* how many will I have for 645£. 17*s.* 6*d*? *Ans.* 1811*lb.* 3*oz.* $13\frac{7}{8}$*dwt.*

Fifth. At 19£. 0*s.* $1\frac{3}{4}$*d.*, the *tun*, what will 186 T. 11 C. 2*qrs.* come to? *Ans.* 3546£. 5*s.* $8\frac{81}{160}$*d.*

CONTRACTIONS.

We have said, (page 40) that the multiplication of complex numbers contains a great number of particular cases, which can be solved by abridged methods; we shall here lay down these cases.

FIRST CASE.

When the multiplier does not surpass 12.

Begin the operation by multiplying the units of the lowest denomination.

EXAMPLES.

147£.	17*s.*	8*d.*	*multiplicands,*	309£.	19*s.*	5*d.*
		7	*multipliers,*			12
1035£.	3*s.*	8*d.*	*Products,*	3719£.	13*s.*	0*d.*

SECOND CASE.

When the multiplier is greater than 12, but the exact product of two factors, both not exceeding 12.

Multiply first by one of the factors, and then the product so found, by the other factor; this last product will be the answer.

EXAMPLES.

Let 547*£.* 7*s.* 9*d. be multiplied by* 27 = 3 × 9

multiply first	547*£.* 7*s.* 9*d.*
by	3
then multiply	1642 - 3 - 3
by	9
	14779*£.* 9*s.* 3*d.* *product of the two given numbers.*

	£. s. d.			*£. s. d.*
multiply	1 - 10 - 8	*by*	44	*Ans.* 67 - 9 - 4.
	4 - 16 - 3½	*by*	56	—— 269 - 12 - 4.
	1 - 13 - 6¾	*by*	121	—— 203 - 1 - 0¾.
	0 - 4 - 9	*by*	144	—— 34 - 4 - 0.

THIRD CASE.

When the multiplier is composed of the product of two factors, more a number which does not exceed 12.*

* It may be easily conceived that the following method is applicable to any multiplier, how great soever it be. Suppose, for instance, that we have 728 for multiplier, this number results from 10 × 10 × 7 + 10 × 2 + 8; Then we should first multiply the multiplicand by 10; this first product being

First, find the product (per last Rule) of the multiplicand by the two factors, then multiply the same multiplicand by the last part of the multiplier, and add the two products together; the sum is evidently the product required.

EXAMPLES.

Let 1£. 17*s*. 4½*d*. *be multiplied by* 23 = 21 + 2 = 3 × 7 + 2.

	£. s. d.	
first multiply	1 - 17 - 4½	
by	3	
then multiply	5 - 12 - 1½	
by	7	
	39 - 4 - 10½	*product by* 21.

£. s. d.		
finally mult. 1 17 4½	3 - 14 - 9	*product by* 2.
by 2	42 - 19 - 7½	*product by* 23.

	£. s. d.				£. s. d.
multiply	3 - 13 - 4	*by*	31	*Ans.*	113 - 13 - 4
	0 - 16 - 6½	*by*	47	——	38 - 17 - 5½
	1 - 18 - 10	*by*	68	——	132 - 0 - 8
	1 - 15 - 4½	*by*	155	——	274 - 3 - 1½

multiplied again by 10, and this fecond by 7, we would obtain a third product, to which fhould be added twice the product of the firft multiplicand by 10, then the product of the fame multiplicand by 8; and the fum of thefe three products would be the final refult of the operation. This method is fometimes ufed in practice, to avoid the calculation by the aliquot parts, which in some cafes may be more complicated, but which cannot be avoided when the multiplier is a complex number.

PRACTICE.

This is the natural place of this article, since by the name of *practice*, are meant rules for the more expeditious solution of questions, depending entirely on Multiplication.

There are none of these questions, but can be solved by the general method given for complex multiplication; It will then suffice to point out the particular cases, in which this method admits of being abridged.

1st. *When the rate of the unit is 1s. to have the price of the whole quantity in pounds and shillings.*

You must take at once the twentieth part of the number expressing the total quantity; which is done by writing the figure of units of that number in the column of shillings, and taking the half of the other figures, which you write in the column of pounds; but, if this half is not a whole number, write 1 on the left of the figure representing the shillings.

Questions.

First. What cost 447 *yards* of ribbon, at 1*s. per yard?* *Ans.* 22£. 7*s.*

Second. What cost 27 *dozen of knives,* at 1*s. per piece?* *Ans.* 16£. 4*s.*

Third. What cost 257 forks at 1*s. per piece.?* *Ans.* 12£. 17*s.*

N. B. We have seen that, *when the rate of a unit is of two shillings, we must take the tenth part of the whole number.*

Secondly, *When the rate of a unit is an even number of shillings greater than 2.*

The shortest method is to multiply the figure of units of the total number by the number of shillings, to count this product as shillings, then to multiply all the other figures by the half of the same number of shillings, and write this product as pounds.

QUESTIONS.

1st. What cost 256 *gallons* of shrub, at 6*s.* *per gallon* ? *Ans.* 76£. 16*s.*

			£.	s.
2nd.	What cost 453 *yds*, at 2*s.* *per yd* ?	*Ans.*	45	6
3d. - - -	984 *lb.* of green tea, at 8*s.* *per lb* ?	——	393	12
4th. - - -	120 C. of beef, at 10*s.* *per C* ?	——	60	0
5th. - - -	427 *reams* of paper, at 12*s.* *per ream* ?	——	256	4
6th. - - -	78 C. of cheese, at 14*s.* *per Cwt* ?	——	54	12
7th. - - -	526 *yds* of broad cloth, at 16*s.* *per yd* ?	——	420	16
8th. - - -	156 C. of rice, at 18*s.* *per Cwt.* ?	——	140	8

Thirdly, *When the rate of the unit is an odd number of shillings greater than 1.*

The solution of this case is obtained by the two preceding rules, taking first the product by one shil-

ling and adding thereto the product by the even number of shillings which remain.

QUESTIONS.

		£.	s.
What cost 184 *yards* of linen, at 3*s.* *per yard*?	*Ans.*	27	12
347 *stones* of wool, at 9*s.* *per stone*?	——	156	3
833 *stones*, at 11*s.* *per stone*?	——	458	3
129 *Cwt.* of Iron, at 13*s.* *per Cwt.*	——	83	17

Fourthly. *Considerable abbreviations can be obtained by the use of the following tables,* which contain the aliquot parts of the *Pound, penny, and hundred weight.*

Other tables, of the same kind, could be formed of other quantities, which admit of it; but these are deemed sufficient as being those of common use in trade.

These tables shew the aliquot parts under a more extensive denomination than has been given in the first part of this work. The learner must commit them to memory, to have them at command when necessity requires.*

We have also given in these tables the *complements* of aliquot parts. *We call complements, what is wanting to these parts to make an integer.* Some Examples will shew the use and utility of complements.

Practice Tables.

Aliquot parts of 1 £.

s.	d.	£.		s.	d.
10 -	0	is $\frac{1}{2}$	complements	10 -	0
6 -	8	- $\frac{1}{3}$	- - -	13 -	4
5 -	0	- $\frac{1}{4}$	- - -	15 -	0
4 -	0	- $\frac{1}{5}$	- - -	16 -	0
3 -	4	- $\frac{1}{6}$	- - -	16 -	8
2 -	6	- $\frac{1}{8}$	- - -	17 -	6
2 -	0	- $\frac{1}{10}$	- - -	18 -	0
1 -	8	- $\frac{1}{12}$	- - -	18 -	4
1 -	0	- $\frac{1}{20}$	- - -	19 -	0

0 -	6	- $\frac{1}{40}$	- - -	19 -	6
0 -	8	- $\frac{1}{30}$	- - -	19 -	4
0 -	4	- $\frac{1}{60}$	- - -	19 -	8
0 -	3	- $\frac{1}{80}$	- - -	19 -	9
0 -	2	- $\frac{1}{120}$	- - -	19 -	10

Aliquot parts of 1*s.*

d.	s.		d.
6	is $\frac{1}{2}$	complements	6
4	- $\frac{1}{3}$	- - - -	8
3	- $\frac{1}{4}$	- - - -	9
2	- $\frac{1}{6}$	- - - -	10
$1\frac{1}{2}$	- $\frac{1}{8}$	- - - -	$10\frac{1}{2}$
1	- $\frac{1}{12}$	- - - -	11

Aliquot parts of a Cwt.

qrs.	*lb.*	*Cwt.*		*qrs.*	*lb.*
2 -	0	is $\frac{1}{2}$	complement	2 -	0
1 -	0 -	$\frac{1}{4}$	- - -	3 -	0
0 -	16 -	$\frac{1}{7}$	- - -	3 -	12
0 -	14 -	$\frac{1}{8}$	- - -	3 -	14
0 -	8 -	$\frac{1}{14}$	- - -	3 -	20
0 -	7 -	$\frac{1}{16}$	- - -	3 -	21

By means of these tables will be solved the following

QUESTIONS.

The rate of the unit being an aliquot part of the pound.

1st. At 6*s*. 8*d*. *per yard*, what is the amount of 337 *yards* ? *Ans*. 112£. 6*s*. 8*d*.

2nd. At 3*s*. 4*d*. *per lb*, what cost 936*lb*. of coffee ? *Ans*. 156£.

3d. At 2*s*. 6*d*. *per lb*, what cost 224*lb* ? *Ans*. 28£.

4th. What cost 1928 *hats*, at 5*s*. *per hat* ? *Ans*. 482£.

5th. At 4*s*. *per pair*, what cost 726 *pair* of shoes ? *Ans*. 145£. 2*s*.

6th. What cost 1755 *pair* of stockings, at 2*s*. *per pair* ? *Ans*. 175£. 10*s*.

7th. At 1*s*. 8*d*. *per yard*, what cost 3127 *yards* of linen-cloth ? *Ans*. 260£. 11*s*. 8*d*.

The rate of the unit being an aliquot part of a shilling.

We give here the process to be followed, which is very convenient, when the rate is at the same time an aliquot part of the pound.

1st. At 6*d.* *per lb.* what cost 512*lb.* of sugar? *Ans.* 12£. 16*s.*

We have taken $\frac{1}{4}$ of 51, which we have counted as pounds; then $\frac{1}{2}$ of 32 which have been set down as shillings.

2d. At 4*d.* *per quart*, what must I give for 504 quarts? *Ans.* 8£. 8*s.*

We have taken $\frac{1}{6}$ of 50, and $\frac{1}{3}$ of 24.

3d. At 3*d.* *per oz.* what come 156 *oz.* to? *Ans.* 1£. 19*s.*

We have taken $\frac{1}{8}$ of 15, and $\frac{1}{4}$ of 76.

4th. At 2*d.* *per yard*, what cost 758 yards of tape? *Ans.* 6£. 6*s.* 4*d.*

We have taken $\frac{1}{12}$ of 75; then $\frac{1}{6}$ of 38, and finally multiplied 2, which remained, by the rate of one yard.

To solve the two following questions, we have first sought, by the aliquot parts, the total price in shillings, then we have taken $\frac{1}{20}$ to reduce it to pounds.

5th. At 1$\frac{1}{2}$*d.* *per lb.* what cost 342*lb*? *Ans.* 2£. 2*s.* 9*d.*

6th. What will 976*lb.* come to at 1*d.* *per lb.*? *Ans.* 4£. 1*s.* 4*d.*

Of the Complements.

1. *When the price is the complement of an aliquot part of* 1£.

Rule.

Find the amount of the aliquot part whereof it is the complement, and subtract the same from the quantity taken as so many pounds; the remainder is the answer.

Application.

Let it be required to find the price of 713*lb.* at 13*s.* 4*d. per lb*?

	lb.		
	713	*s.*	*d.*
13*s.* 4*d.* *Complement* $\frac{1}{3}$	237	13	4
Ans.	475£.	6*s.*	8*d.*

At 15*s. per Cwt.* what cost 336 *cwt.* of logwood? *Ans.* 252£.

How much cost 194 *yards* of broad-cloth, at 16*s.* 8*d. per yard*? *Ans.* 161£. 13*s.* 4*d.*

What cost 479 *yards* of velvet, at 17*s.* 6*d*? *Ans.* 419£. 2*s.* 6*d.*

At 18*s.* 4*d. per Cwt.* what cost 95 *Cwt.* of rice? *Ans.* 87£. 1*s.* 8*d.*

At 19*s. per Cwt.* what cost 317 *Cwt.*? *Ans.* 301£. 3*s.*

Second. *When the price is the complement of an aliquot part of a shilling.*

The same rule as above.

Application.

1st. At 9*d*, *per lb*. What cost 1784*lb* of tobacco?

	1784*s*.
9*d*. complement $\frac{1}{4}$	446
Ans.	1338*s*. or 66£. 18*s*.

2nd. At 10*d*. *per yard*, what cost 540 *yards*? *Ans*. 22£. 10*s*.

3d. How much cost 7644*lb*, at 10$\frac{1}{2}$*d*. per *lb*? *Ans*. 334£. 8*s*. 6*d*.

4th. What is the price of 5649*lb*. of pepper, at 11*d*. *per lb*? *Ans*. 258£. 18*s*. 3*d*.

Division of Complex or Compound Numbers.

Questions.

1st. If 26*fm*. 3*ft*. 4*in*. cost 420£. 4*s*. 10*d*. what was the *rate of a fathom*? *Ans*. 15£. 16*s*. 6*d*.

2d. Paid 547£. 7*s*. 3*d*. for the repair of a road 15 *leagues* 2 *miles* 6 *furlongs* in length; what comes the league to? *Ans*. 34£. 7*s* 9$\frac{81}{191}$*d*.

3d. Bought 27*Cwt*. 3*qrs*. 14*lb*. of sugar, at 257£. 6*s*. 7*d*; what cost 1*Cwt*? *Ans*. 9£. 4*s*. 7$\frac{127}{223}$*d*.

4th. How many *Cwt*. *qrs*. and *lb*. of coffee can I have for the sum of 75£. 8*s*. if the *Cwt*. cost 12£. 4*s*? *Ans*. 6*Cwt*. 0*qrs*. 20$\frac{49}{61}$*lb*.

Contractions.

Contraction in compound Division, takes place only when the Divisor being of one denomination, or re-

duced to it, forms a number, which does not exceed 12, or is the product of two numbers not exceeding 12.

It is easily conceived that the method to be followed in this place is the same which has been employed for shortening the division of incomplex numbers in similar cases.

EXAMPLES.

When the Divisor does not exceed 12.

	£. s. d.			£. s. d.
1st. *Divide*	459 - 14 - 4	*by* 3.	*Ans.*	153 - 4 - $9\frac{1}{3}$
	£. s. d.			£. s. d.
2nd.	3 - 15 - 1	*by* 7.	——	0 - 10 - $8\frac{5}{7}$
	Cwt. qrs. lb.			*Cwt. qrs. lb.*
3d.	153 - 1 - 4	*by* 8.	——	19 - 0 - 18
	Cwt. qrs. lb.			*Cwt. qrs. lb.*
4th.	27 - 1 - 0	*by* 12.	——	2 - 1 - $2\frac{1}{3}$

When the Divisor is composed of two factors not exceeding 12.

	£. s. d.			£. s. d.
1st. *Divide*	102 - 14 - 6	*by* 21.	*Ans.*	4 - 17 - 10
2nd.	27 - 16 - 0	*by* 32.	——	0 - 17 - $4\frac{1}{2}$
3d.	269 - 12 - 4	*by* 56.	——	4 - 16 - $3\frac{1}{2}$
4th.	202 - 13 - 6	*by* 121.	——	1 - 13 - 6

QUESTIONS.

1st. A piece of broad cloth containing 24 *yards*, cost 18£. 6*s*; what did it cost *per yard*? *Ans.* 15*s*. 3*d*.

2nd. Bought 49 *Cwt.* of butter for 343 £. 8*s.* 4*d.* what did it cost *per Cwt.*? *Ans.* 7 £. 0*s.* $2\frac{2}{49}$*d.*

3d. If a hogshead of wine cost 16£. 16*s*; what was it a gallon? *Ans.* 5*s.* 4*d.*

4th. If 27*Cwt.* of sugar cost 47£. 12*s.* $10\frac{1}{2}$*d.* what cost 1*Cwt*? *Ans.* 1£. 15*s.* $3\frac{1}{2}$*d.*

5th. If I hold 120 *acres* of land, and my yearly rent is 87£; what do I pay an *acre*? *Ans.* 14*s.* 6*d.*

6th. If a reckoning of 6£. 14*s.* 2*d.* is to be paid by 35 persons, what must they pay a piece? *Ans.* 3*s.* 10*d.*

ALLIGATION.

Questions.—First Case.

1st. Suppose 15 *bushels* of wheat, at 9*s.* the *bushel*, and 12 *bushels* of rye, at 6*s.* 4*d.* *the bushel*, be mixed together; what is the mean rate or price it may be sold at a *bushel*, without loss or gain? *Ans.* 7*s.* $9\frac{7}{9}$*d.*

2nd. A tobacconist mixes 36*lb.* of tobacco, worth 1*s.* 6*d.* *a pound*, with 12*lb.* of another sort at 2*s.* *a pound*; and 12*lb.* of a third sort at 1*s.* 10*d.* *per pound*; how must he sell the mixture *per pound*? *Ans.* 1*s.* 8*d.*

3d. A vintner mixes $31\frac{1}{2}$ *gallons* of Malaga, worth 7*s.* 6*d.* the *gallon*; with 18 *gallons* of Canary, at 6*s.* 9*d.* the *gallon*; $13\frac{1}{2}$ *gallons* of Sherry, at 5*s.* the *gallon*; and 27 *gallons* of White-wine at 4*s.* 3*d.* the *gallon*. It is required to find what *one gallon* of this mixture is worth? *Ans.* 6*s.* *per gallon*.

4th. There are melted and mixed together, two sorts of silver; one sort is worth 5*s*. and the other 4*s*. an ounce; and there are 4 *ounces* of the first, and 8 *ounces* of the latter; what is the value of 1 *ounce* of this mixture? *Ans.* 4*s*. 4*d*.

5th. A gold-smith melts 8*lb*. $5\frac{1}{2}$*oz*. of gold bullion of 14 *caracts** fine with 12*lb*. $8\frac{1}{2}$*oz*. of 18 caracts fine. How many caracts fine is the mixture? *Ans.* $16\frac{51}{127}$ caracts.

SECOND CASE.

1st. How much rye at 6*s*. *the bushel*, and barley at 4*s*. 2*d*. *per bushel*, will make a mixture that may stand in 5*s*. 3*d*. *the bushel*? *Ans.* 13 of rye, and 9 of barley.

2nd. What quantity of sugar at 11*d*. the *lb*. and $7\frac{1}{4}$*d*. the *lb*. would make a mixture such that it would stand in $10\frac{1}{4}$*d*. the *lb*? *Ans.* 3*lb*. at 11*d*. and $\frac{3}{4}$*lb*. at $7\frac{1}{4}$*d*.

3d. How much rye at 4*s*. *per bushel*, barley at 3*s*. *per bushel*, and oats at 2*s*. *per bushel*, will make a mixture worth 2*s*. 6*d*. *per bushel*? *Ans.* 6 *bushels* of rye, 6 *bushels* of barley, and 24 *bushels* of oats.

4th. A vintner would make a mixture of Malaga, worth 7*s*. 6*d*. *per gallon*, with Canary at 6*s*. 9*d*. *per gallon*, Sherry at 5*s*. *per gallon*, and White-wine at 4*s*.

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* An ounce of pure gold being reduced into 24 equal parts, these parts are called *caracts*; but gold is often mixed with some baser metal, which in the mixture is called the *alloy*; and according to the proportion of pure gold, which is in every ounce, so the mixture is said to be so many caracts fine; thus if only 22 caracts of pure gold, and 3 of alloy, it is 22 caracts fine.

3*d.* *per gallon* ; what quantity of each must he take, that the mixture may be sold for 6*s.* *per gallon* ? *Ans.* 12 of Malaga ; 18 of Sherry ; 21 Canary ; and 9 White-wine.

It is easy to see, that, when there are more than three quantities, the problem has several solutions.

5th. How much wheat at 5*s.* the *bushel*, must be mixed with 12 *bushels* of rye, at 3*s.* 6*d.* *per bushel*, that the whole mixture may be at 4*s.* 4*d.* *per bushel* ? *Ans.* 15 *bushels.*

6th. How much brass of 14*d.* *per lb.* and pewter of $10\frac{1}{2}$*d.* the *lb.* must I melt with 50*lb.* of copper, worth 16*d.* the *lb.* so that the whole may stand me in 1*s.* the *lb* ? *Ans.* 200 at $10\frac{1}{2}$*d.* and 50 at 14*d.*

7th. With 60 *gallons* of brandy at 6*s.* *per gallon*, I mix brandy of 5*s.* 4*d.* *per gallon*, and some water ; then I find it stood me in 3*s.* 6*d.* *per gallon* ; I demand how much brandy, and water I took ? *Ans.* 60 at 5*s.* 4*d.* and $74\frac{2}{7}$ of water.

To solve questions of the same kind with the following, you must, after using the common rule, multiply each of the numbers found, by the quotient of the total quantity of the mixture demanded, divided by the sum of the numbers found. This process is founded upon the same principles with the observations inserted at the bottom of page 55.

8th. A Vintner has three sorts of wine, *viz.* of 24*d.* 22*d.* and 18*d.* *the gallon* ; now he has a mind to mix a cask of 60 *gallons*, so that he may sell it at 20*d.* *the gallon* ; how much must he take of each ? *Ans.* 12 at 24*d.* 12 at 22*d.* and 36 at 18*d.*

9th. A grocer has sugar of 12*d.* the *lb.* and $6\frac{1}{4}$*d.* the *lb.* and has a mind to mix 2*cwt.* of it, so that he may sell it at 8*d.* the *lb.* I demand how much of each sort he must take? *Ans.* $162\frac{10}{11}$*lb.* of $6\frac{1}{4}$*d.* and $61\frac{1}{11}$ of 12*d.*

10th. A cask of 60 *gallons* is filled with liquor of 7, 8, and 10*d. the gallon*, and then it stands in $9\frac{1}{4}$*d. the gallon*; I wish to know how many *gallons* of each sort were taken? *Ans.* 42 *gallons of* 10*d.* 9 *of* 8*d.* *and* 9 *of* 7*d.*

RULE OF THREE SIMPLE.

Questions.

1st. What do 518*lb.* of tea come to, if 90*lb.* cost 18£. *Ans.* 103£. 12*s.*

2nd. How many *Cwt.* of sugar may be bought for 27£. when the price of 40*cwt.* is 150£? *Ans.* 7*cwt.* 0*qrs.* $22\frac{2}{5}$*lb.*

3d. What will the tax upon 763£. be, at the rate of 50*s.* for 12 Pounds? *Ans.* 158£. 19*s.* 2*d.*

4th. A draper bought 242 *yards* of broad cloth for 254£. 10*s.* for 86 yards of which he gave 21*s.* 4*d. per yard*; what was the price *per yard* of the remainder. *Ans.* 20*s.* $10\frac{16}{39}$*d.*

5th. If 5 *yards* of cloth cost 14*s.* 2*d.* what must be given for 9 pieces containing each 21 *yards* 1*qr.* *Ans.* 27£. 1*s.* $10\frac{1}{2}$*d.*

6th. If a man's annual income be 500£. and he expend daily 19*s.* 11*d*; what does he save at the end of the year? *Ans.* 136£. 10*s.* 5*d.*

7th. If a staff, 4 feet long, cast a shade (on level ground) 7 feet; what is the height of that steeple, whose shade, at the same time, measures 198 feet? *Ans.* $113\frac{1}{7}$*ft.*

8th. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far are the inhabitants at the equator carried in one minute, a degree there being $69\frac{1}{2}$ *miles*? *Ans.* 17*m.* 3*fur.*

With Fractions.

9th. If $\frac{4}{7}$ of an ell cost $\frac{7}{13}$£. What is that per ell? *Ans.* 18*s.* $10\frac{2}{13}$*d.*

10th. If $\frac{1}{2}\frac{1}{3}$*lb.* of sugar cost $\frac{7}{12}$*s.*; what cost $\frac{3}{4}\frac{2}{3}$*lb*? *Ans.* 4*d.* 3 $\frac{16}{23}\frac{57}{65}$*qrs.*

11th. If $6\frac{1}{8}$ *yards* cost 18*s*; what buys $9\frac{1}{4}$ *yards*? *Ans.* 1£. 5*s.* 7*d.* $1\frac{7}{13}$*qrs.*

12th. If $1\frac{1}{4}$ *yard* cost 9*s*; what is the value of $16\frac{1}{4}$ *yards*? *Ans.* 5£. 17*s.*

RULE OF THREE COMPOUND.

1st. If the carriage of 8 *Cwt.* 128 *miles* cost 48 shillings, for how much can I have 4 *Cwt.* carried 32 miles, after the same rate? *Ans.* 6*s.*

2nd. If 240£. in 16 months gain 64£. how much will 60£. gain in 6 months? *Ans.* 6£.

3d. With how many pounds sterling, could I gain 5£. per annum, if with 450£. I gain in 16 months 30£? *Ans.* 100£.

4th. If 1 pound of thread make 3 yards of linen 5 quarters broad, how many pounds of thread would be wanted to make a piece of linen 45 yards long, and 1 yard broad? *Ans.* 12 *pounds.*

5th. If a footman perform a journey in 3 days, when the days are 16 hours long; how many days 12 hours long, will he take to perform the same journey? *Ans.* 4 *days.*

6th. How many yards of canvass, that is ell wide, will be sufficient to line 20 yards of say, that is 3*qrs.* wide? *Ans.* 15 *yards.*

7th. Suppose 800 soldiers were placed in a garrison, and their provisions were computed sufficient for two months; how many soldiers must depart, that the provisions may serve them 5 months? *Ans.* 480 *men.*

8th. A master of a vessel having provisions for 30 days for a crew of 16 men, falls in with a boat with four unfortunate shipwrecked men, whom he takes on board; he wishes to know how many days his provisions will last him with this addition of people? *Ans.* 24 *days.*

9th. A master of a vessel having 9 men on board, finds that his provisions will just do for 28 days, which he calculates his voyage may last; he picks up three lost men, and then asks what part of the *ration* he must give to each man on board that his provisions may last him the same time? *Ans. the* $\frac{3}{4}$ *of a ration.*

10th. If 80000 *Cwt.* of ammunition were to be removed from a place in 9 days, and in 6 days

time I find 4500 *Cwt.* carried away by 18 horses; how many horses would be wanted to carry away the remainder in 3 days? *Ans.* 604 *horses.*

11th. If 12 men in 8 days gain 8£. 8*s.* what will 21 men gain in 15 days? *Ans.* 27£. 11*s.* 3*d.*

12th. How many days will it take for 72 men to dig a canal 645 yards long, by 20 feet wide and 8 feet deep, by working 12 hours a day, if 6 men have dug a canal 75 yards long 10 feet wide, and 6 feet deep, in 36 days, working 10 hours a day? *Ans.* $57\frac{1}{3}$ *days.*

With Fractions.

13th. If 72 men are $57\frac{1}{3}$ days to accomplish a work 645 yards long, 20 feet broad and 8 feet deep, by working 12 hours a day, how many hours a day must 6 men work to make one 75 yards long, 10 feet broad, and 6 feet deep in the space of 36 days? *Ans.* 10 *hours.*

14th. How many yards of lining $\frac{3}{4}$ broad, will it take to line a cloak containing $5\frac{1}{3}$ yards of a cloth $\frac{5}{8}$ broad? *Ans.* $4\frac{4}{9}$ *yards.*

15th. 36 men, in 24 days, at 12 hours a day, have made 240 yards of a cloth $\frac{5}{8}$ broad: How many yards of a cloth $\frac{3}{4}$ broad will 80 men make in $75\frac{3}{5}$ days, working 16 hours per day? *Ans.* 1120 *yards.*

16th. If $\frac{3}{4}$ yard of cloth that is $\frac{7}{8}$ yard wide, cost $\frac{2}{3}$£. what is the value of $\frac{5}{6}$ yard, that is $1\frac{3}{4}$ yard wide, being of the same quality? *Ans.* 13*s.* 4*d.*

FELLOWSHIP SIMPLE.

Questions.

1st. A and B buy certain merchandizes, amounting to 80£. of which A pays 30£. and B 50£. they gain by said goods 20£. now I would know each man's share in the profit, in proportion to the sum he puts in? *Ans.* A's 7£. 10*s.* B's 12£. 10*s.*

2nd, A merchant being deceased, it is found he owes to A 500£. and to B 900, though he left but 1100£. behind him; I demand how much each is to have in proportion to his debt? *Ans.* A 392$\frac{6}{7}$ and B 707$\frac{1}{7}$.

2d. A, B, and C, freighted a ship with 108 tuns of wine, of which A had 50 tuns, B 34, and C 24; but, by reason of stormy weather, the mariners were obliged to cast 45 tuns overboard; how much must each man sustain of the loss? *Ans.* A 20T. 3*hhd.* 21*gal.* B 14T. 0*hhd.* 42*gal.* C 10T.

4th. A merchant has been bankrupt in $.900,000, and leaves to his creditors only $.36000; what proportion must each of them receive, supposing that

to the 1st. is due	30000	*Ans.*	1st.	1200
to the 2nd.	16000		2nd.	640
to the 3d.	82000		3d.	3280
to the 4th.	66500		4th.	2660
to the 5th.	12500		5th.	500
to the 6th.	693000		6th.	27720

5th. Four Booksellers have undertaken the publication of a work, which costs them $.2500, and of

which they have struck off 6000 copies.—Query: how many copies is each entitled to, from the various sums they have laid out in the undertaking, which are as follow:

the 1st. has given	\$.850	Ans.	1st. 2040	Copies.
2nd. - - -	530		2nd. 1272	
3d. - - -	785		3d. 1884	
4th. - - -	335		4th. 804	

FELLOWSHIP COMPOUND.

1st. Three merchants have made a company, the first has put in 4000£. for 12 months, the second 3000 for 15 months, and the third 5000 for 8 months, and they gain 665£. What is each man's part in the profits? *Ans.* the first 240£. the second 225, and the third 200.

2nd. Three graziers have hired a piece of land for 60£. 10*s*. the first has put in 5 oxen for $4\frac{1}{2}$ months, the second 8 oxen for 5 months; and the third 9 oxen for $6\frac{1}{2}$ months; I demand how much each pays? *Ans.* The first 11£. 5*s*. the second 20£. and the third 29£. 5*s*.

3d. A, B, and C, have made a stock for $2\frac{1}{2}$ years, A has put in at first 6000£. and 2 years after 4000 more; B has put in at first 12000£. and at the end of the eight months he has taken out the whole, but 12 months after he has put in 7000; C has put in 5000£. which have remained all the time in trade; the total gain is found to be 5200£. what is each man's share thereof? *Ans.* A 2040£. B 1660, C 1500.

4th. A, B, and C, formed a company, and gained in 22 months the sum of 3600£. A put in at first

6000£. and a year after put in again 2000 ; B put in 4000£. but 18 months after he put in 4000 more ; C put in at first 9875£. but 16 months after he withdrew 8875 ; what is each man's share of the gain ? *Ans.* A 1302£. 17*s*. $1\frac{5}{7}$*d*. B 891£. 8*s*. $6\frac{6}{7}$*d*. and C 1405£. 14*s*. $3\frac{3}{7}$*d*.

5th. A, B, and C, join stocks for 12 months ; A puts in 100£. and the first of the fifth month 150 more, and on the first of the ninth month takes out 30 ; B puts in 250£, on the first of the sixth month 60 more, and on the first of the eleventh month 100 more ; C puts in 300£, on the first of the fourth month takes out 200, and on the first of the eighth month takes out 50 more ; the whole gain is 133£. What is each partner's share of it ? *Ans.* A must have 40£. 14*s*. $0\frac{600}{745}$*d*, B 64£. 12*s*. $6\frac{90}{745}$*d*, and C 27£. 13*s*. $5\frac{555}{745}$*d*.

RULE OF INTEREST.

1st. Case.

The general method to solve the questions in that case, is (without attempting any reduction in the proportion, as the division by 100 is very easy,) to multiply the given sum by the interest, and divide the product by 100, which is performed as follows :

Questions.

1st. What is the interest of 270£. 10*s*. 6*d*. at the rate of 5 per cent, per annum ?

W 2

£. s. d.	*Proportion.*
270 - 10 - - 6	£. £. £. s. d.
5	100 : 5 :: 270 - 10 - 6 : x.

£. 13 | 52 - 12 - - 6
20

s. 10 | 52
12

d. 6 | 30
4

qr. 1 | 20

Ans. 13£. 10s. 6¼*d.**

2nd. What is the interest of 344£. 17*s.* 6*d.* for 1 year, at the rate of 6 per cent. per annum? *Ans.* 20£. 13*s.* 10*d.*

3d. What is the interest of 246£. 18*s.* 10*d.* for one year, at 3 per cent. per annum? *Ans.* 7£. 8*s.* 1¾*d.*

4th. What is the interest of 220£. for one year, at 4 per cent. per annum? *Ans.* 8£. 16*s.*

5th. What is the interest of 300£. 10*s.* for one year, at the rate of 8 per cent. per annum? *Ans.* 24£. 0*s.* 9½*d.*

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* We have neglected the fraction, as it falls under one *fourth* of a penny. This appears to be generally received in trade,† in order to avoid the trouble of calculating fractions of so small a value; yet it seems that accuracy would require that every fraction of *half a fourth* of a penny and *above*, should be counted as a *fourth* of a penny, and that those only should be omitted, which are below *half a fourth*.

† Vide American Tutor's Assistant.

SECOND CASE.

Questions.

1st. What is the interest of 176£. 13s. 9*d*. at the end of 9 years, at 5 per cent. per annum? *Ans.* 79£. 10*s*. $2\frac{1}{4}$*d*.

2nd. Calculate the interest of a bond for 427£. 18*s*. 9*d*. *for* 2 years, at $5\frac{3}{4}$* per cent, per annum? *Ans.* 49£. 4*s*. 3*d*.

3d. What is the principal and interest, or *amount*, of 1096£. 15*s*. 6*d*. for 4 years, at $6\frac{1}{2}$ per cent. per annum? *Ans.* 1381£. 18*s*. 8*d*.

4th. What is the interest of 428£. 14*s*. for 9 months, at 4 per cent. per annum? *Ans.* 12£. 17*s*. $2\frac{1}{2}$*d*.

5th. What is the interest of 57£. 17*s*. 8*d*. for three months, at 6 per cent. per annum? *Ans.* 17*s*. $4\frac{1}{4}$*d*.

6th. Find the interest of 1000£. for one year and nine months, at 8 per cent. per annum? *Ans.* 140£.

7th. How much is the interest of 175£. 10*s*. 6*d*. for 1 year and 7 months, at 6 per cent. per annum? 16£. 13*s*. $5\frac{3}{4}$*d*.

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* When the rate per cent. is a *mixt number*, find out separately, by the given method, the intereſt for the principal units and then for the fraction, which you muſt add up together.

8th. Cast up the interest of 886£. 16*s.* for 3 years and $11\frac{1}{2}$ months, at 4 per cent. per annum. *Ans.* 140£. 8*s.* $2\frac{1}{4}$*d.*

9th. What is the interest of 456£. 15*s.* 6*d.* for 5 years, 8 months, and 23 days at 7 per cent?

We here give the detail of the calculation, in order to point out a correction, which must be made in the method given in the first part, to find with greater exactness the interest for a certain number of days. Counting, as we have done in the first part, the month at 30 days, the year would have only 360 days instead of 365. If then we had an interest of 6£. for one year, the interest for one day would be exactly expressed by $\frac{6}{365}$, whilst in the supposition of 30 days per month, this same interest would be $\frac{6}{360}$. These two interests are then to one another as $\frac{6}{365}$: $\frac{6}{360}$, or as $\frac{1}{73}$: $\frac{1}{72}$, or as 1 : $\frac{73}{72}$; whereby we see that the second exceeds the first by $\frac{1}{72}$. The correction must then be made in this manner.

Begin the operation, as has been said, by multiplying the given sum by the interest. This product should be divided by 100 for the interest of one year. But this division is put off until the end of the calculation.

Now, taking up the question,

	£.	s.	d.	
We have	456 ..	15 ..	6	
to be multiplied by			7	
1 year -	3197 ..	8 ..	6	interest for one year
4 years	12789 ..	14 ..	0	
4 months	1065 ..	16 ..	2	
4 do. -	1065 ..	16 ..	2	
20 days -	177 ..	12 ..	$8\frac{1}{4}$	*20 days are the sixth of 4 months.*
2 do. - -	17 ..	15 ..	3	
1 do.	8 ..	17 ..	$7\frac{1}{2}$	
£. -	183|23 ..	0 ..	$4\frac{3}{4}$	

	20
s. - -	4|60
	12
d. - -	7|24
	4
qr. - -	0|99

	£.	s.	d.
	183 ..	4 ..	7
to be subtracted			6
Ans.	183 ..	4 ..	1

The 6d. subtracted make the correction spoken of, they proceed from the total sum of pounds, for the interest of the 23 days; this sum is of 202£. of which striking off the units, we have taken the third of the 20 *tens*, and count them as pence, which is the same as if we had taken the 7200th part of 202.*

* For in the firſt place by taking the third of the *tens*, we divide them by 3. 2dly. If we counted them for units of pounds, they would then be thrice ten times or 30 times ſmaller; then to count them as ſhillings, they muſt again be divided by 20, which makes them twenty times thirty times, or ſix hundred times ſmaller; In fine, to be brought to pence, this laſt value muſt be divided by 12, which makes them 7200 times ſmaller.

But this number is to be divided by 100; it is then 100 times too great; the result obtained by this division is then equal to the one we should have by taking the 72nd part of the true number. Thus, for instance, the 72nd part of 36, which is $\frac{1}{2}$, is the same with the 7200th part of 3600, which is 100 times greater than 36.

Whenever then you have an interest to find out for a number of days, reduce this number into months of 30 days, and parts thereof. But forget not to make the above mentioned correction.

10th. What is the interest, at 4 per cent, of 300£. from the 12th of June exclusively, to the 15th of February inclusively.

	£.		
	300		
	4		
	1200 £. interest for 1 year.		
6 months - - -	600		
2 do. - - - - -	200		
6 days - - - -	20		
2 do. - - - - - -	6	.. 13s .. 4d.	
	8|26	.. 13 .. 4	
	20		£. s. d.
	5|33		8 .. 5 .. 4
	12		*to sub.* 2 .. 3
	4|00		8 .. 3 .. 1

The interest for a certain number of days can also be found by the compound rule of three; for, the preceding question can be expressed thus:

If 100£. during 365 days, give 4£. of interest, how much will give 300£. during 248 days? *then we have the proportion* $100 \times 365 : 4£. :: 300 \times 248 : x = 8£. - 3s. - 0\frac{3}{4}d.$

This result differs a little from the preceding, because in the correction we have paid no regard to the 72nd part of the units of pounds shillings and pence, which ought to have been also subtracted, to render the correction perfectly exact; but this difference is so trifling, that the first method is often preferred to the second, as being shorter, especially when the interest comprises, at the same time, years, months, and days, or only months and days.

Proof.

There is another method, besides the one just given, which in many cases will perhaps be more expeditious, and in all cases may serve to prove the former. It is as follows:

When the interest of any sum, for any time, is at 6 per cent. per annum. *Multiply the principal by half the time in months, and divide by* 100, (because the interest for 2 months is 1;) if these be days, *take for them such part or parts of the principal as half the days are of* 30; *making for the interest so found, the correction above mentioned.* If the days exceed 30, *bring them into months of* 30 *days each, and make the correction for the total.*

After having found the interest at 6 *per cent. increase or diminish it by proportional parts thereof for any other rate:* as,

for	1 per cent. take			$\frac{1}{6}$	of the interest at 6 per cent.
	2	-	- - -	$\frac{1}{3}$	
	3	-	- - -	$\frac{1}{2}$	-
	4	-	*subtract*	$\frac{1}{3}$	*of*
	5	-	*subtract*	$\frac{1}{6}$	*of*
	7	-	- *add*	$\frac{1}{6}$	*to said interest*
	&c.				

11th. What is the interest of 827£. 18*s*. 10½*d*. for 1 year 11 months and 20 days, at 6 per cent. per annum?

M.	D.			£.	s.	d.
23	20			827	18	10½
						11½
11½	10 half time	M.				
		11	-	9107	7	7½
		½	-	413	19	5¼
		D.				
		10		275	19	7½
		£.		97|97	6	8¼
				20		
		s.		19|46		
				12		
		d.		5,60		
				4		
		qr.		2|41		

	£.	s.	d.
	97	19	5¼
			9
Ans.	97	18	8½

12th. What sum will 674£. 13*s*. 8¾*d*. amount to in 5 years 11 months and 28 days, at 6 per cent. per annum? *Ans.* 917£. 6*s*. 1½*d*.

13th. What is the interest of 517£. 15*s*. 4*d*. for one month, at 6 per cent. per annum? *Ans.* 2£. 11*s*. 9¼*d*.

14th. What is the interest of 327£. 10s. at 6 per cent. per annum, for 210 days? *Ans.* 11£. 6s. 1d.

15th. Tell the interest of 240£. for one year and 135 days, at 7 per cent. per annum? *Ans.* 23£. 0s. 3d.

16th. What is the interest of 371£. for 1 year and 213 days, at 5 per cent. per annum? *Ans.* 29£. 7s. 6d.

17th. What is the interest of a bond for 325£. 15s. 6d. for 1 year and 73 days, at 4 per cent. per annum? *Ans.* 15£. 12s. 9d.

18th. What sum will a bond of 435£. 13s. $5\frac{3}{4}d$. amount to in two years 8 months and 19 days, at 3 per cent. per annum? *Ans.* 471£. 4s. 2d.

19th. What interest is due on a legacy of 517£. 12s. $8\frac{1}{4}d$. for 3 years 11 months and 26 days, at 2 per cent per annum? *Ans.* 41£. 5s. $8\frac{1}{2}d$.

CONTRACTIONS.

When the time is any number of years, of months separately or collectively, and the rate at 5 per cent. per annum.

Take $\frac{1}{20}$ of the principal for the interest for 1 year.

The reason of this process is obvious, since we have the proportion, 100 : 5 :: *the principal* : *the interest for one year*, and making the reduction 20 : 1 :: *the principal* : &c. Having in this manner obtained the interest for one year, you will, by the given method, find it for any number of years and months.

20th. What is the interest of 573£. 16*s*. 9*d*. for 5 years, at 5 per cent. per annum? *Ans.* 143£. 9*s*. 2*d*.

21st. What is the interest of 435£. 8*s*. 6*d*. for 7 months, at 5 per cent. per annum. *Ans.* 12£. 13*s*. 11¾*d*.

22nd. What is the interest of a bond for 326£. 13*s*. 5*d*. for 3 years and ten months, at 5 per cent. per annum? *Ans.* 62£. 12*s*. 2½*d*.

This compendious method may serve to find the interest at any rate per cent, by either diminishing or augmenting proportionably, the interest at 5 per cent. as follows,

for interest at	2½	*per cent. take*	½	*of*	*the interest at 5 per cent.*
	3	- - - -	½ + $\frac{1}{10}$	*of*	
	3½	- - - -	½ + ⅕	*of*	
	4	- *subtract*	⅕	*of*	
	4½	- - - -	$\frac{1}{10}$	*of*	
	5½	- *add* -	$\frac{1}{10}$	*to*	
	6	- - - -	⅕	*to*	
	6½	- - -	⅕ + $\frac{1}{10}$	*to*	
	7	- - - -	⅕ + ⅕	*to*	

Observe that the interest for a number of days cannot be found by this method, as it does not afford the same facility for making the correction above mentioned.

Questions relative to the Rule of Interest.

First Case.

To find the principal, when the amount, time, and rate per cent are given.

RULE.

Make that proportion (as already said in the first part,)

As the amount of 100, *at the rate and time given, is to* 100£. *so is the amount given, to the principal required.*

EXAMPLES.

1st. What principal will amount to 725£. at interest for 9 years, at 5 per cent. per annum? *Ans.* 500£.

2nd. What sum at interest for 9 years and 6 months, and at $4\frac{1}{2}$ per cent. per annum, will amount to 856£. 10s? *Ans.* 600£.

SECOND CASE.

To find the rate per cent, when the amount, time and principal are given.

RULE.

Subtract the principal from the amount, to have the whole interest, and then make the following proportion.

As the principal is to the interest of the whole time, so is 100£. *to its interest for the same time.*

Finally divide the interest last found by the time, and the quotient will be the rate per cent.

EXAMPLES.

1st. At what rate per cent per annum, will 500£. amount to 725£. in 9 years? *Ans. 5 per cent.*

2nd. At what rate per cent will 600£. amount to 856£. 10s. in 9 years and 6 months? *Ans. 4½ per cent.*

Third Case.

To find the time, when the principal, amount, and rate per cent are given.

Rule.

Subtract the principal from the amount, and divide the whole interest so found, by *that* of the principal *for one year*, and the quotient will be the time required.

Examples.

1st. in what time will 500£. amount to 725£. at 5 per cent. per annum? *Ans. 9 years.*

2nd. in What time will 600£. amount to 856£. 10s. at 4½ per cent per annum? *Ans. 9 years 6 months.*

Fourth Case.

Insurance, Commission and Brokage.

Insurance, commission and brokage, are allowances made to insurers, factors, or brokers, at a stipulated rate per cent.

Rule.

For the insurance or commission, the premium of which is a certain number of pounds per cent, work as if to find the interest of the given sum, at the proposed rate for one year; and for the brokage which is rated commonly at less than one pound per cent,

divide the sum by 100, and take such aliquot parts of the quotient, as the brokage is of a pound.

Examples.

1st. A factor has disbursed, upon his employer's account, the sum of 1009£. 18s. what must be demanded for his commission, at $2\frac{1}{4}$ per cent? *Ans.* 22£. 14s. $5\frac{1}{4}d$.

2nd. What is the insurance of an East-India ship and cargo, valued at 7406£. 17s. 6d. at $15\frac{3}{4}$ per cent. *Ans.* 1166£. 11s. $7\frac{1}{2}d$.

3d. Suppose $1\frac{3}{4}$ per cent be allowed for commission, what must be demanded on 704£. 15s. 4d? *Ans.* 12£. 6s. 8d.

4th. What is the brokage of 700£. 14s. 6d. at 4s. per cent? *Ans.* 1£. 8s. $0\frac{1}{4}d$.

5th. What may a Broker demand on 420£. 12s. 6d. at 6s. 4d. per cent? *Ans.* 1£. 6s. $7\frac{1}{2}d$.

DISCOUNT, OR REBATE.

Discount, or Rebate, is an abatement for the payment of money before due.

Questions.

1st. What is the discount of 161£. 10s. for 19 months, at 5 per cent. per annum? *Ans.* 11£. 16s. $11\frac{1}{4}d$.

2nd. What is the rebate of 795£. 11s. 2d. for 11 months, at 6 per cent. per annum? *Ans.* 41£. 9s. $5\frac{3}{4}d$.

3d. Sold goods for 795£. 11s. 2d. to be paid 4 months hence; what is the present worth, the rebate at $3\frac{1}{2}$ per cent. per annum? *Ans.* 786£. 7s. 8d.

4th. What is the discount of a bill of exchange of 112£. 12s. for 20 months, at 7 per cent. per annum? *Ans.* 11£. 15s. $3\frac{1}{4}$d.

5th. Sold goods for 312£; one half to be paid at 3 months, and the other half at 6 months; what must be discounted for present payment at 5 per cent. per annum? *Ans.* 5£. 14s. 7d.

6th. What is the present worth of 100£. one half payable at 4 months, and the other at 8 months; discount at 5 per cent. per annum? *Ans.* 97£. 11s. $3\frac{3}{4}$d.

FALSE POSITION.

Questions.

1st. Four men have a sum to divide, on condition that the first will have $\frac{1}{3}$, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth will have the remainder, amounting to 28£. What is the sum to be divided?

Solution.

I suppose a number, of which I can easily obtain the third, the fourth, and the sixth; 12 can be this number, of which $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$, make up 9, and 3 remains; therefore, I will have this proportion 3 (remainder of 12) : 12 :: 28 : the demanded sum = 112£.

PROOF.

Share of the 1st. the $\frac{1}{3}$	- -	37£.	- 6s.	- 8d.
2nd. $\frac{1}{4}$	- -	28	- - 0	- 0
3d. $\frac{1}{6}$	- -	18	- - 13	- 4
4th. the remainder		28	- - 0	- 0
		112	- - 0	- 0

2nd. Find a number, of which the $\frac{1}{3}$, the $\frac{1}{2}$, and the $\frac{1}{4}$, make 52.

SOLUTION.

I take 12, of which the $\frac{1}{3}$, the $\frac{1}{4}$, and the $\frac{1}{2}$ make 13; now I have this proportion, $13 : 12 :: 52 :$ the number asked $= 48$.

3d. A man dying, leaves a sum of 36000£. to be divided among five persons, in the following manner; he gives to the first $\frac{1}{4}$ of the sum, to the second $\frac{1}{3}$, to the third, $\frac{1}{6}$, to the fourth $\frac{1}{12}$, to the fifth $\frac{2}{3}$, what will be each man's share?

SOLUTION.

It must be observed that the several legacies exceed the estate left, since $\frac{1}{4} + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{2}{3}$, make more than 1; but, as their parts must be in the same ratio with these fractions, the question must be solved in this manner.

Let us suppose that the whole sum is 12£. of which $\frac{1}{4} + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{2}{3}$, make 18£. then we have this proportion; $18 : 12 :: 36000 : x = 24000$; —now dividing this number conformably with the intention of the testator, we easily find the several person's shares, the sum of which really amounts to 36000£.

the 1st. has	6000£. the	$\frac{1}{4}$	of 24000£.
2nd.	8000 - -	$\frac{1}{3}$	
3d.	4000 - -	$\frac{1}{6}$	
4th.	2000 - -	$\frac{1}{12}$	
5th.	16000 - -	$\frac{2}{3}$	
	36000£.		

SQUARE ROOT.

Examples.

The square root of - - - 529 - *is* - 23
of - - 2025 - - - 45
of - - 14641 - - 121
of 11744329 - 3427
of 25280784 - 5028
of 9042049 - 3007
of $(6+\frac{1}{4})=\frac{25}{4}$ *is* $\frac{5}{2}$

Decimal Numbers.

The square root of 2,89 - *is* - 1,7
of 43,5* - - - 6,5 *remainder* 125
of 1,356 - - - 1,16 - - - 104
of $\frac{2}{3}$ *within a tenth, is* 0,8
of $\frac{5}{7}$ *within a hundredth* 0,84
of 5,7 *within a tenth, is* 2,3
of 2,34, *within a hundredth* 1,52

* Put a cypher at the right of the 5, in order to have an even number of decimals.

CUBE ROOT.

TABLE.

Roots	1	2	3	4	5	6	7	8	9
Cubes.	1	8	27	64	125	216	343	512	729

EXAMPLES.

The cube root of - - 10648 - *is* - 22
of - - 1728 - - - 12
of 13997521 - - - 241
of 6983587 - - - 403
of $(3 + \frac{3}{8}) = \frac{27}{8}$ - *is* - $\frac{3}{2}$

DECIMAL NUMBERS.

The cube root of 15,625 - *is* - 2,5
of 1,8 - - - 1,2 *remainder* 72
of 2,97 - - 1,4 - - - 226
of $\frac{2}{3}$ *within a tenth, is* 0,8
of $\frac{6}{7}$ *within a hundredth* 0,94
of 6,2 *within a tenth, is* 1,8

TARE AND TRETT.

Tare and trett are allowances made by the seller to the buyer, on some particular commodities.

Tare is the weight of the *barrel, box, bag*, or whatever contains the goods.

Trett is an allowance for the *waste* and *dust*, of 4lb. in every 104lb.

Gross is the weight of the goods, together with that in which they are contained.

Neat is the weight of the goods, after all allowances are deducted.

First Case.

Find the value of goods, allowances done for *tare* only.

Rule.

It is evident that we must subtract the whole tare from the whole gross weight, and multiply the difference, *which is the neat*, by the price, as taught for compound multiplication.

Questions.

1st. At 1£. 10s. 6d. per cent. what is the value of 24 hogsheads of tobacco, each weighing 6C. 2qr. 17lb. *gross weight*, and the whole tare being 17C. 3qr. 27lb? *Ans.* 216£. 0s. 4½d.

2nd. At 2£. 14s. per cent. what is the value of 12 casks of raisins, each weighing 3C. 2qr. 10lb. *gross*, and *tare* 20lb. per cask? *Ans.* 110£. 10s. 1½d.

3d. Sold 9 hogsheads of sugar, each 6C. 2qr. 12lb. *gross*, *tare* 17lb. per hundred weight: what is the neat weight, and what sum must be paid at 2£. 12s. 6d. per *Cwt.*? *Ans. neat* 50C. 1qr. 22lb.* amount 132£ 8s. 5¼d.

* In ſuch queſtions as theſe, the ounces are neglected.

Second Case.

When the trett is allowed with tare.

Rule.

Deduct the tare as before; the remainder is called *suttle*, which divided by 26,* the quotient will be the *trett*; subtract this from the suttle, and the remainder will be the *neat*, which, multiplied by the price, gives the answer.

Questions.

1st. In 27 bags of coffee, each 2C. 3*qr*. 17*lb*. *gross*, *tare* 13*lb*. per *Cwt*; *trett* 4*lb*. per 104*lb*; what is the *neat*, and what is its value at 3£. 18*s*. 9d. per *Cwt*.? *Ans*. *Neat* 66C. 2*qrs*. 11*lb*. *value* 262£. 4*s*. 7$\frac{1}{4}$*d*.

2nd. At 8$\frac{1}{2}$*d*. per pound, what is the value of the neat weight of 8C. 3*qrs*. 20*lb*. *gross*, *tare* 38*lb*, *trett* 4*lb*. in every 104*lb*.? *Ans*. 32£. 15*s*. 2$\frac{1}{2}$*d*.

BARTER.

Barter is the exchanging of one commodity for another, by duly proportioning their quantities and values.

From this definition, it follows that the questions relative to barter, depend upon proportions, and are solved by the rule of three.

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* 4*lb*. in every 104*lb*. is 1*lb*. in evey 26*lb*.

Questions.

1st. How much sugar, at 9*d.* per *lb.* could be bartered for $6\frac{1}{4}$ *Cwt.* of tobacco, at 14*d.* per *lb.*?

It is plain that the $6\frac{1}{4}$ *Cwt.* of tobacco, at 14*d.* per. *lb.* must produce the same sum as the quantity of sugar demanded (which I call x) at 9*d.* per *lb.* considering then the causes and effects, we form the proportion, $6\frac{1}{4}C. \times 14d. : 1 :: x\,C. \times 9d. : 1$, and $x = 10C.$ $0qr.\ 12\frac{4}{9}lb.$

Note. You must carefully observe that the compared quantities are to be of the same kind. The following question will shew the necessity of this remark, and point out the operation to be performed, when the two quantities are of different kinds.

2nd. How much rice, at 28*s.* per *Cwt.* must be bartered for $3\frac{1}{2}$C. of raisins at 5*d.* per *lb*?

You may work it in two ways. 1st find the price of a *lb.* of rice expressed in cents, or 2dly. find in shillings and parts of shillings, the price of a *Cwt.* of raisins, which will give the two following proportions, both leading to the same result.

$$\begin{array}{llllllll} & C. & d. & C. & d. & C. & qr. & lb. \\ \text{1st.} & 3\frac{1}{2} & \times\, 5 : 1 :: & x & \times\, 3 : 1\ \&\ x = & 5 & - 3 & - 9\frac{1}{3} \end{array}$$

$$\begin{array}{llllllll} & C. & s.\ d. & & s. & C. & qr. & lb. \\ \text{2nd.} & 3\frac{1}{2} & \times(46-8) : 1 :: & x & \times 28 : 1\ \&\ x = & 5 & - 3 & - 9\frac{1}{3} \end{array}$$

The result is much more speedily come at by the first than by the second proportion;—Practice will discover the shortest way in the different cases.

3d. How much sugar at 8*d.* per *lb.* must be delivered for 20C. of tobacco at 3£. per *Cwt*? *Ans.* 16C. 0*qrs.* 8*lb.*

4th. A gives B 250 yards of drugget at 18½*d.* per yard, for 308⅓*lb.* of pepper: I demand what the pepper stands him in per *lb*? *Ans.* 15*d.*

5th. A has linen cloth worth 20*d.* an ell ready money, but in barter he will have 2*s*; B has broad cloth worth 14*s.* 6*d.* per yard, ready money; at what price ought the broad cloth to be rated in barter? *Ans.* 17*s.* 4⅕*d.*

6th. A has 320 dozen of candles at 9*s.* per dozen, for which B agrees to pay him 60£. in cash, and the rest in cotton at 16*d.* per *lb.* how much cotton must B give A? *Ans.* 1260*lb.*

7th. C has candles at 12*s.* per dozen, ready money, but in barter he will have 13*s.* per dozen; D has cotton at 18*d.* per *lb.* ready money; what price must the cotton be in barter, and how much must be bartered for 100 dozen of candles? *Ans. The cotton at* 19½*d. per lb. and* 800*lb. must be given for* 100 *dozen of candles.*

8th. A barters 40 yards of cloth at 7*s.* 4*d.* per yard, with B, for 28½*lb.* of tea, at 11*s.* 6*d.* per *lb.* which must pay balance, and how much? *Ans.* A 1£. 14*s.* 5*d.*

EXCHANGE.

Payments being commonly made by letters of exchange from one country to another, the rule of exchange, which is nothing else than a rule of three, has for its principal object to determine *from the*

course of exchange,* the value which a sum received in one country, must have in another.

Having only in view to make the rule known, we shall confine its application to the exchange between the state of Maryland and England; But in order to understand the following questions, it must be observed, that the pound in these two countries, though subdivided into an equal number of shillings and pence, yet has not the same real value;—100 English pounds amount to 166⅔ pounds, Maryland currency; this ratio is what they call the *Par*;—The course of exchange is seldom at that rate, being almost always either higher or lower; and, supposing it to be at 163 or 168, it is said to be simply 63 or 68.

QUESTIONS.

1st. Baltimore is indebted to London 1474£. 16s. currency; what sterling sum must be remitted when the exchange is at 64 per cent.

It is evident that we have the proportion 164 : 100 :: 1474£. 16s : *the answer* 899£. 5s. 4¼d.

2nd. London receives a bill of exchange from Baltimore for 943£. 17s. 5¼d. sterling; for how much currency was it drawn, exchange being at 64 per cent? Here we have the proportion 100 : 164 :: 943 £. 17s. 5¼d : *The answer* 1547£. 18s. 11⅛d.

* To convey a perfect knowledge of every circumstance incidental to exchange, it would require a detail rather long; a verbal explication from the professor will easily make up any deficiency.

There is a compendious and ready method of finding out in such a case the fourth term of the proportion.

Divide the second term by the first, and multiply the third by the quotient, the division is easily performed, as below, by considering the second term of the proportion as composed of cents and aliquot parts of cents; we have then here

$$164 = \left\{\begin{array}{r}100\\50\\10\\2\\2\end{array}\right\} \begin{array}{c}\text{each part be-}\\\text{ing divided}\\\text{by 100, give}\\\text{for the}\\\text{quotients}\end{array} \left\{\begin{array}{c}1\\\frac{1}{2}\\\text{\&c.}\\\\\\\end{array}\right\}$$

then the multiplication is done by the aliquot parts, as follows:

Product for	100	943£.	17s.	5¼d.
	50	471	18	8½
	10	94	7	8¾
	2	18	17	6½
	2	18	17	6½
	Ans.	1547£.	18s.	11½d.

3d. How much sterling is equal to 1341£. 9s. 4¾d. Maryland currency, exchange at 67½ per cent? *Ans.* 800£. 17s. 6½d.

4th. In a settlement between C of Baltimore and D of London, C is indebted 750£. 2s. 4 d. sterling, what sum Baltimore currency is equivalent, exchange at 78 per cent? *Ans.* 1335£. 4s. 2¼d.

Contraction.

When exchange is *at par*, then the questions are very easily solved; because we find that 166⅔ are

exactly the $\frac{2}{3}$ above 100;—Then to reduce sterling to Maryland currency, it suffices to add to the given sum, the $\frac{2}{3}$ of the same; and on the contrary to bring Maryland currency into sterling money, you must take the $\frac{3}{5}$ of the given sum; for, 100 are exactly the $\frac{3}{5}$ of 166$\frac{2}{3}$.

Questions.

5th. What sum, Baltimore currency, will be equal to 345£. 13*s.* 6*d.* sterling, exchange at 66$\frac{2}{3}$? *Ans.* 576£. 2*s.* 6*d. currency.*

6th. What sum sterling will be equal to 736£. 3*s.* 9*d.* Baltimore currency, exchange at 66$\frac{2}{3}$? *Ans.* 441£. 14*s.* 3*d. sterling.*

RULES AND PROBLEMS OF

ALGEBRA.

MULTIPLICATIONS.

Products.

1st. $\left\{\begin{array}{ll}\text{Multiply} & a+b\\ \text{by} & a+b\end{array}\right\} a^2+2ab+b^2$

2nd. $\left\{\begin{array}{ll}\text{Multiply} & a-b\\ \text{by} & a-b\end{array}\right\} a^2-2ab+b^2$

3d. $\left\{\begin{array}{l}a+b\\ a-b\end{array}\right\} a^2-b^2$

4 th. $\left\{\begin{array}{r}2a-3\\ a+2\end{array}\right\} 2a^2+a-b$

5th. $\left\{\begin{array}{l}4a^2-6a+9\\ 2a+3\end{array}\right\} 8a^3+27$

6th. $\left\{\begin{array}{l}a^2+2ab+2b^2\\ a^2-2ab+2b^2\end{array}\right\} a^4+4b^4$

7th. $\left\{\begin{array}{l}2a^2-3ab-4b^2\\ 3a^2+2ab+b^2\end{array}\right\} \begin{array}{l}6a^4-13a^3b-4a^2\\ b^2+5ab^3-4b^4\end{array}$

8th. $\left\{\begin{array}{l}a^3+2a^2b+2ab^2+b^3\\ a^3-2a^2b+2ab^2-b^3\end{array}\right\} a^6-b^6$

9th. $\left\{\begin{array}{l}a^2-ab+b^2-ac-bc+c^2\\ a+b+c\end{array}\right\} \begin{array}{l}a^3-3abc\\ +b^3+c^3\end{array}$

DIVISIONS.

			Quotients.
1st.	Divide	$a^2 + 3ab + 2b^2$	$a + 2b$
	by	$a + b$	
2nd.	Dd	$a^3 - 3a^2b + 3ab^2 - b^3$	$a^2 - 2ab + b^2$
	by	$a - b$	
3d.	Dd	$18a^2 - 8b^2$	$6a + 4b$
	by	$3a - 2b$	
4th.	Dd	$6a^2b^2 - 4ab^3 + a^4 + b^4 - 4a^3b$	$a^2 - 2ab + b^2$
	by	$b^2 + a^2 - 2ab$	
5th.	Dd	$a^4 + 4a^2b^2 + 16b^4$	$a^2 - 2ab + 4b^2$
	by	$2ab + 4b^2 + a^2$	
6th.	Dd	$a^4 + 4b^4$	$a^2 + 2ab + 2b^2$
	by	$a^2 - 2ab + 2b^2$	
7th	Dd	$1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$	$-x^3 + 3x^2 - 3x + 1$
	by	$1 - 2x + x^2$	

NOTE. The proofs of the different multiplications, can serve as examples for the division, and reciprocally.

FRACTIONS.

Examples, *to reduce Fractions to the same denominator.*

1st. $$\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{4ab}{a^2-b^2}$$

2nd. $\dfrac{a}{b-a}-\dfrac{b-a}{a-2b}=-\dfrac{b^2}{3ab-a^2-2b^2}=\dfrac{b^2}{a^2+2b^2-3ab}$

3d. $-\dfrac{a-b}{a}+\dfrac{b}{a-b}-\dfrac{a+b}{b}=\dfrac{-a^2b+4ab^2-b^3-a^3}{a^2b-ab^2}$

4th $\dfrac{3a^2}{a-2b}-\dfrac{a-2b}{a}+\dfrac{a}{a+2b}=\dfrac{3a^4+6a^3b+4ab^2-8b^3}{a^3-4ab^2}$

5th $\dfrac{a-2b}{2b-1}-\dfrac{3b-1}{1-3a}=\dfrac{a-3a^2+6ab-6b^2+3b-1}{2b-1-6ab+3a}$

6th, $\dfrac{3a-b}{a-2b}-\dfrac{2+3a}{a-4b}=\dfrac{-7ab+4b^2-2a+4b}{a^2-6ab+8b^2}$

RADICAL QUANTITIES.

Examples of Radical Quantities put under an exponential form.

1st. $\sqrt{a\,.\,b^2\,c^3\,d^4}=a^{\frac{1}{2}}\,b\,.\,c^{\frac{3}{2}}\,d^2$

2nd. $\sqrt[3]{a^3\,b^2\,c^6\,d}=a\,b^{\frac{2}{3}}\,c^2\,d^{\frac{1}{3}}$

3d. $\sqrt[3]{a^{\frac{1}{2}}\,b^3\,c^{\frac{2}{3}}}=a^{\frac{1}{6}}\,b\,c^{\frac{2}{9}}$

4th. $\sqrt[2]{a^{\frac{3}{2}}\,b^{\frac{2}{3}}\,c\,d^2}=a^{\frac{3}{4}}\,b^{\frac{1}{3}}c^{\frac{1}{2}}\,d$

Examples *of Radical Quantities simplified as much as possible.*

1st. $\sqrt{36 a^5 b^4 c d^3} = 6 a^2 b^2 d \sqrt{a c d}$

2nd. $\sqrt{40 a^3 b c^2 d} = 2 a c \sqrt{10 a b d}$

3d. $\sqrt[3]{54 a^6 b^2 c^3 d^4} = 3 a^2 c d \sqrt[3]{2 b^2 d}$

4th. $\sqrt[3]{56 a^4 b^3 c^6 d} = 2 a b c^2 \sqrt[3]{7 a d}$

INVOLUTION.

Examples *for raising a Binomial to different powers.*

1st. $(a+b)^3 = a^3+3 a^2 b+3 a b^2+b^3$

2nd. $(a-b)^3 = a^3-3 a^2 b+3 a b^2-b^3$

3d. $(a+b)^4 = a^4+4 a^3 b+6 a^2 b^2+4 a b^3+b^4$

4th. $(a-b)^6 = a^6-6 a^5 b+15 a^4 b^2-20 a^3 b^3+15 a^2 b^4-6 a b^5+b^6$

EQUATIONS.

FIRST DEGREE, WITH ONE UNKNOWN QUANTITY.

Examples *of Equations solved.*

1st. $ax+b-c=d$, gives $ax=d-b+c$, and $x=\dfrac{d-b+c}{a}$

2nd. $\frac{x}{a}+b-c=d\ldots\frac{x+ab-ac}{a}=d\ldots x+ab-$

$ac=ad\ldots$ *and* $x=ad-ab+ac$

3d. $\frac{x}{3}-1+2a=3+a\ldots\frac{x-3+6a}{3}=3+a\ldots$

$x-3+6a=9+3a\ldots x=12-3a$

4th. $\frac{x}{a-1}-1=a\ldots\frac{x-a+1}{a-1}=a\ldots x=a^2-1$

5th. $\frac{2x}{3}-4=1\ldots\frac{2x-12}{3}=1\ldots 2x=3+12\ldots x=\frac{15}{2}$

6th. $\frac{3x}{4}+\frac{1}{2}=5\ldots\frac{3x+2}{4}=5\ldots 3x=20-2\ldots x=\frac{18}{3}=6$

7th. $x+\frac{x}{2}+5=11\ldots\frac{2x+x}{2}=6\ldots 3x=12\ldots x=4$

8th. $x+\frac{x}{2}+\frac{x}{3}=44\ldots\frac{6x+3x+2x}{6}=44\ldots x=24$

9th. $ax-bx+cx=d\ldots(a-b+c)x=d\ldots x=\frac{d}{a-b+c}$

10th. $3x+2=10+x \ldots 3x-x=10-2 \ldots$

$2x=8 \ldots x=4$

11th. $x+8=32-3x \ldots 4x=24 \ldots x=6$

12th. $1+x=5-\frac{x}{2} \ldots 1+x=\frac{10-x}{2} \ldots 2+$

$2x=10-x \ldots x=\frac{8}{3}$

13th. $\frac{1}{2}-\frac{x}{3}=\frac{1}{3}-\frac{x}{4} \ldots \frac{3-2x}{6}=\frac{4-3x}{12}$

$\ldots 6-4x=4-3x$, *and* $x=2$

14th. $1\frac{1}{2}-\frac{2x}{3}=\frac{1}{4}+\frac{x}{2} \ldots \frac{3}{2}-\frac{2x}{3}=\frac{1}{4}+\frac{x}{2}$,

and $\frac{9-4x}{6}=\frac{1+2x}{4} \ldots 18-8x=3+6x \ldots x=\frac{15}{14}$

15th. $\frac{100}{x}-8=12 \ldots 100-8x=12x \ldots x=5$

16th. $\frac{5x+3}{x-1}=7 \ldots 5x+3=7x-7 \ldots x=5$

17th. $\frac{x}{1-\frac{1}{4}} - \frac{x-11}{4} = \frac{5x}{3} + 4 \ldots 2x - \frac{x-11}{4} =$

$\frac{5x+12}{3} \ldots \frac{8x-x+11}{4} = \frac{5x+12}{3} \ldots 21x+33$

$= 20x+48 \ldots x = 15$

NOTE. Some of these equations might have been solved in a more expeditious manner, but we have chosen to adhere to one uniform method, which is applicable to all cases, in order both to engrave it the more deeply into the minds of beginners, and to avoid perplexing them with particular methods, which practice alone will soon make them discover.

PROBLEMS OF THE FIRST DEGREE,

WITH ONE UNKNOWN QUANTITY.

First. Divide 54£. among three persons, so that the share of the second be double that of the first, and the third lot be equal to both the others; *What will be each person's share?*

NOTE. The question seems to involve three unknown quantities, but a little reflection will shew, that the first person's share being known, the other two will of course be also known. This observation deserves particular attention, as it is applicable to a great number of problems, the enunciation of which seems to suppose more unknown quantities than there

really are. Be x the share of the first person; then we will have the equation $x + 2x + 3x = 54 \ldots 6x = 54 \ldots x = 9$.

Second. Divide 112£. among three persons, so that the second have three times the share of the third, and the first 3 times the share of the second and third together. *What will be the share of each.* Be x the share of the third person; we will have $x = 7$.

Third. Four Merchants have a sum of $.58000 to divide in the following manner: The second is to have $\frac{1}{2}$ the share of the first, the third $\frac{2}{3}$ of the second's, the ourth $\frac{3}{4}$ of the third's; *Say the share of each.* Be x the share of the first; we shall have the equation,

$$x + \frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 58000, \text{ and } x = \$.27840.$$

Fourth. A father dying leaves an estate of 15000£. to be distributed to his widow, a son and a daughter, as follows: the daughter to have only $\frac{2}{3}$ of the son's share, and the mother $\frac{1}{4}$ of the sum of both the children's portions; *what sum will each receive?* Be x the share of the son, the equation will be,

$$x + \frac{2x}{3} + \frac{1}{4}\left(x + \frac{2x}{3}\right) = 15000£. \ldots x = 7200£.$$

Fifth. Divide 720 in three parts, the greatest of which exceed the smallest by 80, and the mean one by 40. *Ans.* 280 ... 240 ... and 200.

Sixth. How old are we, asks a son to his father; your age, replies his father, is now the third of mine,

and six years ago it was but the fourth of it. *Find the ages of both*; *Ans.* The father's 54 years, and the son's 18.

Seventh. Divide 25 in two parts, such that their difference divided by 3, be equal to 5.

Be x one of the parts, $25 - x$ will be the other; then we will have the equation, $\frac{x - (25 - x)}{3} = 5$

$\therefore x - 25 + x = 15 \ldots x = 20.$

Eighth. Divide 20 in two parts, so that the third of the greater be equal to the half of the smaller. *Ans.* 12 and 8.

Ninth. Divide 45 in two parts, which be to one another as 5 is to 4. *Ans.* 25 and 20.

Tenth. A courier sets off from Baltimore, and runs 9 miles an hour; another courier sets off 5 hours after, on the same road, and runs 12 miles an hour; *At what distance from Baltimore will the second overtake the first.*

NOTE. The problems on *couriers*, which generally appear very perplexing to beginners, can all be solved in the same manner, by means of a proportion, which it is always very easy to establish; this proportion is founded upon the remark, that the spaces run by the two couriers in the same time, are evidently in the same ratio as their respective velocities; for instance, if one runs twice as fast as another, he will of course go twice the length of the other in the same time.

Be x the distance demanded in the present problem, it is plain that during the time the second courier goes x, the first will go $x - 45$; these 45 representing the space performed by the first in 5 hours start, which he has over the second, at the rate of 9 miles an hour: The proportion will then be, $12 : 9 :: x : x - 45$ ... and $x = 180$ *miles*.

Eleventh. A courier starts from Baltimore for New-York, and goes 12 miles an hour; another dispatched for the same city, starts from Philadelphia 3 hours after the first has left Baltimore, and goes 6 miles an hour. *At what distance from Baltimore will they meet, supposing Philadelphia is* 102 *miles distant from Baltimore.*

Be x the distance demanded.

Observe that the first courier has set off three hours before the second, and has consequently run 36 miles before the departure of this; that, therefore, the second has only $x - 102$ to run in the same time the first will run $x - 36$. The proportion will then be, $12 : 6$, or $2 : 1 :: x - 36 : x - 102$ . . . and $x = 168$.

Twelfth. Two couriers set off at the same moment, the one from Baltimore, the other from Philadelphia, and direct their course towards each other. It is supposed that the one from Baltimore goes at the rate of 8 miles, and that from Philadelphia of 10 miles an hour. *At what distance from Baltimore will they meet?* 102 miles is the distance of these two cities from each other. *Ans.* $45\frac{1}{3}$ *miles.*

Thirteenth. A courier sets off from Philadelphia for Boston, and runs 7 miles an hour; another four hours after leaves Baltimore, who goes at a double rate. *At what distance from Philadelphia will the second overtake the first? Ans* 158 *miles.*

Fourteenth. A man has hired a lazy journeyman, at 24*d.* for each day he would work, but upon condition that he would strike off from his wages 6*d.* for every day he would not work. At the end of 30 days, they draw his account, and find that he has nothing to claim. *How many days has he worked? Ans.* 6.

Fifteenth. A merchant takes out every year, from his stock in trade, for his house expences, a sum of $.1000; but, what remains in trade, increasing by one third every year, he finds at the expiration of three years, that his original stock is doubled. Query: *What was that stock? Ans.* $.14800.

Sixteenth. A father and his son wish to know how many days they would be to finish a work together, which the father alone would perform in 5 days, and the son alone in $7\frac{1}{2}$. *Ans.* 3 *days.*

Seventeenth. Some workmen join to make a certain work, for which they are paid 10 dollars a day; two of them happening to fall sick, the others find themselves able to perform the same daily task, and the price of their day's work is increased by one third. *It is required to know what was at first the number of men, and the compensation they received per day? Ans.* 8 workmen, at 125 cents.

Eighteenth. Two streams have filled a bason in 10 hours, and it is known that one of the streams fills it

in 16 hours. *In how many hours could the other fill it?* *Ans.* 26 hours, 40 minutes.

FIRST DEGREE,

WITH TWO UNKNOWN QUANTITIES.

Equations solved.

EXAMPLES.

1st. $\begin{cases} 3x + 7y = 33 \; \ldots \; 6x + 14y = 66 \\ 2x + 5y = 23 \; \ldots \; 6x + 15y = 69 \end{cases}$

subtracting the first from the second, we find . $15y - 14y = 69 - 66$
whence we draw $y = 3$. Substituting this value of y in the first equation, we have $3x + 7 \times 3 = 33$ . . . and $x = 4$. Substituting for the proof, the values of x and y in the second equation, we find $2 \times 4 + 5 \times 3 = 23$, equation identical.

2nd. $\begin{cases} 4x - 3y = 14 \; \ldots \; 12x - 9y = 42 \\ 8y - 3x = 1 \; \ldots \; 32y - 12x = 4 \end{cases}$

adding up the two equations $32y - 9y = 42 + 4$ . . . . . . whence we draw $y = 2$. Substituting this value of y in the first equation, it becomes, $4x - 3 \times 2 = 14$ . . . and $x = 5$. substituting, for the proof, the values of x and y in the second equation, we have $8 \times 2 - 3 \times 5 = 1$.

Equations to be solved.

EXAMPLES.

1st. $\begin{cases} 12x + 5y = 46 \\ 3x + 9y = 27 \end{cases}$ *we find* $x = 3$, and $y = 2$

NOTE. Observe that the calculation for the exterminating of x, is here abridged; because, multiplying all the terms of the second equation by 4, x is found to have the same coefficient in both equations. This abbreviation can always be made, when one of the coefficients of the unknown quantity, which is to be exterminated, is an exact multiple of the other, or when both have a common factor; and, as it greatly reduces the work, it ought not to be neglected.

2nd. $\left\{\begin{array}{l} 6y - 21x = 9 \\ 7x + 17y = 92 \end{array}\right\}$ *we find* $x = 1$ ... & $y = 5$

NOTE. Here you give to x the same coefficient in both equations, by multiplying all the terms of the second equation by 3.

3d. $\left\{\begin{array}{l} \dfrac{3y}{2} + \dfrac{7x}{5} = 13 \\ \dfrac{4x}{3} - y = \dfrac{8}{3} \end{array}\right\}$ *we find* $x = 5$ ... and $y = 4$

NOTE. You must begin by clearing away the denominators, and then you find that the coefficient of x in both equations has a common factor, which is 2.

4th. $\left\{\begin{array}{l} \dfrac{x}{2} - 2 = \dfrac{y}{3 - \frac{1}{2}} \\ \dfrac{27 - 3x}{1 + \frac{1}{3}} = 7 - \dfrac{11}{4} \end{array}\right\}$ *we find* $x = 8$ ... & $y = 5$

NOTE. Begin always by clearing away the denominators; see page 127, where we have given an example of an equation, with a fractionary denominator.

PROBLEMS OF THE FIRST DEGREE,

WITH TWO UNKNOWN QUANTITIES.

1st. Knowing the sum a of two quantities, and their difference b, *find each of these two quantities.*

Be x and y these two quantities, the two equations will be $x+y=a$ . . . *and* $x-y=b$ . . . we find $x=\frac{a+b}{2}$ *and* $y=\frac{a-b}{2}$.*

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* It has been faid (page 130,) that of the three methods, which can be made ufe of for the elimination, the one we employed was the moft expeditious, *and was applicable to all cafes*; which, however, is true only for the equations of the firft degree; for, in the fuperior degrees, there may be occafion for the other two methods, which we are going to illuftrate, by applying them to the fimple equations, the folution of which we have juft given.

The one of thofe methods is *fubftitution*. Draw from one of the equations the value of one of the unknown quantities, and fubftitute this value in the other equation, which thereby will contain but one unknown. Thus, in the propofed example, the fecond equation gives $x=b+y$; fubftituting this value in the firft, we fhall have $b+y+y=a$; *equation which contains only one unknown quantity.*

The other method confifts in taking the value of the fame unknown quantity in all the equations, and then to equalize thefe different values Here, for example, wifhing to eliminate x, we fhall have from the firft equation $x=a-y$, and from the fecond $x=b+y$; which will give $a-y=b+y$; *equation containing but one unknown quantity.*

2nd. A two story house is 35 feet high—the first story is four feet higher than the second. *What is the elevation of each story?* *Ans.* First story $19\frac{1}{2}$ *feet*—the second $15\frac{1}{2}$ *feet.*

3d. A person having counters in both his hands, takes one from the right to add it to those in the left, and then finds that both hands have an equal number—if this person had passed two from the left into the right, this last hand would have double the number of the other. *How many counters had he in each hand?* *Ans.* 10 and 8.

4th. A merchant has two sorts of coffee; the one at 30 cents, the other at 18 cents per *lb.* He is asked 150*lb.* for $.36. *How many pounds must he take of each sort, to satisfy that demand, and what will be the value of the mixture per lb?* *Ans.* 75*lb.* of each kind—24 *cents the lb.* of the mixture.

5th. A Journeyman has received $.24 for 12 days work, during 8 of which he had his wife and son working with him. At another time he has been employed by the same person during 9 days, during 4 of which he was assisted by his wife and son, and has received for that time $.15. *What did he earn a day, and what was allowed to his wife and son together,* *Ans.* $.1, and $1\frac{1}{2}$.

6th. $.8,82 cents is the price of a mixture composed of 15 bottles of red wine, and 12 of white; and, it is known, that 5 bottles of the white wine are worth as much as 6 of the red. *What is the price of each kind per bottle?* *Ans.* red 30 *cents*—white 36 *cents.*

FIRST DEGREE,

WITH THREE UNKNOWN QUANTITIES.

Equations to be solved.

EXAMPLES.

1st. $$\left.\begin{cases} 4x+3y-3z=1 \\ 2z+6x-5y=2 \\ 4y-2x-z=3 \end{cases}\right\} \text{we find} \begin{cases} x=1 \\ y=2 \\ z=3 \end{cases}$$

2nd. $$\left.\begin{cases} 2z-4y+x=3 \\ 3x-z-4y=3 \\ 6z+2y-4x=2 \end{cases}\right\} \text{we find} \begin{cases} x=3 \\ y=1 \\ z=2 \end{cases}$$

3d. $$\left.\begin{cases} x+5y+7z=163 \\ 3x+3y-2z=24 \\ 2x-y+3z=51 \end{cases}\right\} \text{we find} \begin{cases} x=8 \\ y=10 \\ z=15 \end{cases}$$

To shorten the calculation, suppose $163=a$, $24=b$, $51=d$.

4th. $$\left.\begin{cases} x-\dfrac{y}{2}+\dfrac{z}{3}=5 \\[2ex] y-\dfrac{2x}{5}+\dfrac{z}{2}=5 \\[2ex] 5y-3x=5 \end{cases}\right\} \text{we find} \begin{cases} x=5 \\[2ex] y=4 \\[2ex] z=6 \end{cases}$$

NOTE. Begin always by clearing the terms of their denominators, and observe, that the 3d equation containing only two unknown quantities, the calculation is simplified; since, by the combination of the two first, we shall form another equation, containing

only the two unknown quantities, which are found in the third.

FIRST DEGREE.

PROBLEMS WITH THREE UNKNOWN QUANTITIES.

1st. Three persons have given alms to a beggar, and his whole receipt is 24 cents. It happens, that the first person has given as much as the other two together; and, that his donation exceeds that of the second, by the same quantity by which the second surpasses the third. *What has been the gift of each?* *Ans.* 1st, 12 *cts.*—2nd, 8 *cts.*—3d, 4 *cts.*

2nd. The father of a young mathematician having bought wines of three different qualities, and having taken 9 bottles of the first, 15 of the second, and 12 of the third, his son asks him, what was the total expence, and the price of the first quality. The father answered, if I had taken an equal number of each quality, I would have paid 180*s*? and, the prices of the different qualities are such, that the sum of the greatest and the smallest, is equal to double the price of the mean quality, and that four times the smallest is worth the price of the two others. *What were the prices demanded?* *Ans.* Total expence 174*s.*—price of the first quality 7*s.*

3d. Three gamesters, A, B, C, begin with a certain sum and play three games. In the first B and C play against A, who loses as much money as the other two together have—in the second A and C join against B, who likewise loses a sum equal to that possessed by both his antagonists—and in the third game, A and B being united against C, this one loses likewise

as much money as the other two together possess.—Then these three persons count their money, and find that they have each $.16. *What has been the gain or loss of each?* *Ans.* A has lost $.10—B has won 2—and C has won 8.

SECOND DEGREE.

PROBLEMS.

1st. *Find a number, the square of which, added to twice this same number, make up 24.*

SOLUTION.

Be x that number, we shall have the equation
$x^2 + 2x = 24$ . . . Completing the square we have
$x^2 + 2x + 1 = 1 + 24$ . . . Extracting the square root of each member we find
$x + 1 = \pm\sqrt{25} = \pm 5$ . . . . finally
$x = -1 \pm 5$ . . . . . . Thus on account of the double sign we have . . . .
1st. $x = 4$ . . . 2ndly, $x = -6$

For the proof, by putting successively these two values in the equation of the problem, we have 1st. $16 + 8 = 24$ . . . 2nd, $36 - 12 = 24$.

2nd. *Find a number, the square of which surpasses 4, by a quantity equal to three times that same number.*

SOLUTION.

Be x that number. The equation will be . . . . .
$x^2 - 4 = 3x$ . . . . Transposing we have
$x^2 - 3x = 4$ . . . . Completing the square
$x^2 - 3x + (\frac{3}{2})^2 = (\frac{3}{2})^2 + 4$ . . Extracting the root
$x - \frac{3}{2} = \pm\sqrt{\frac{9}{4} + 4} = \pm\sqrt{\frac{25}{4}} = \pm\frac{5}{2}$ . . . . finally
$x = \frac{3}{2} \pm \frac{5}{2}$ . . . . whence we draw
1st. $x = 4$ . . , 2ndly, $x = -1$

For the proof we have, 1st. $16 - 4 = 12$ . . . 2ndly, $1 - 4 = -3$.

3d. *What number is it, the product of which by 5, added to 11, give the same sum as the excess of three times its square over 1?*

SOLUTION.

Be x that number, the equation will be

$5x + 11 = 3x^2 - 1$ Transposing and permuting

$3x^2 - 5x = 12$ Clearing x^2 of its coefficient

$x^2 - \frac{5x}{3} = 4$ Completing the square

$x^2 - \frac{5x}{3} + (\frac{5}{6})^2 = (\frac{5}{6})^2 + 4$ Whence

$x - \frac{5}{6} = \pm\sqrt{\frac{25}{36} + 4} = \pm\sqrt{\frac{169}{36}} = \pm\frac{13}{6}$ finally

$x = \frac{5}{6} \pm \frac{13}{6}$ whence we draw

1st. $x = 3$ . . . 2ndly, $x = -\frac{4}{3}$

For the proof we have, 1st. $15 + 11 = 27 - 1$ 2ndly, $-\frac{20}{3} + 11 = \frac{48}{9} - 1$, or bringing to the same denominator, $-60 + 99 = 48 - 9$.

4th. Divide 22 into two parts, such that their product make 105. *Ans.* 15 and 7.

5th. Six dozen eggs have cost as many pence as eggs could have been got for 8*d.* *What was the price of the dozen.* *Ans.* 24*d.*

6th. Several persons, solidary the ones for the others, owe a sum of 342£. Three of them being insolvent, the others find themselves obliged to pay each 19£. above what they ought to have paid. *How many debtors were there in all?* *Ans.* 9.

7th. Twenty persons, men and women, meet in the same inn; the men spend $.12 and the women

Suppose b is the first sum placed at interest, t the time of the loan, p the principal and interest of one dollar for one year.

It is evident, that, if one dollar gives p for one year, p will give p^2; for we have this proportion, $1 : p :: p : p^2$; then p^2 is the principal and interests, or the whole sum due for one dollar, at the end of the 2nd. year. It will be p^3 at the end of the 3d. year; and therefore p^t at the end of a number t of years.

Now, if we multiply this sum due for one dollar, by the number b of dollars, we shall have bp for the expression of the sum desired, the logarithm of which will be equal to $Lb + t\ Lp$. It remains only to express the value of each letter. That of p will be found by this proportion, $100 : 106 :: 1 : p = \frac{106}{100}$; thus, by substituting, we have $L\,10000 + 12\ L\,(\frac{106}{100})$; the calculation being made, shews that the ward must receive the sum of \$.20122 and some cents.

3d. A guardian being able to spare the sum of \$.200 every year, on the revenue of his ward, places it at 6 per cent interest, and adds to it the successive interests accruing every year. It is asked, *what sum will result from all these joined sums, for the ward, at the end of his 21 years?* It is supposed, he was but one year old when his guardianship commenced.

SOLUTION.

Let b equal 200; p the principal and interests of one dollar for one year; then b will be the sum due at the end of the second year of the child's age; $b+bp$, the sum due at the end of the third year; $b+(b+bp)\,p$, or $b+bp+bp^2$, the sum due at the end of the fourth year; whence it is seen, that at the end of the 21st. year, the sum due will be, $b+bp+bp^2+bp^3\ \ldots + bp^{19}$; this quantity

shows a geometrical progression, now to be valued.

Let us take the general formula $S = \frac{qx - a}{q - 1}$, we have

$q = p$, $x = bp^{19}$, $a = b$; then $S = \frac{pbp^{19} - b}{p - 1} = \frac{b(p^{20} - 1)}{p - 1}$;

substituting the value of the letters, we have,

$$S = \frac{200\left[\left(\frac{106}{100}\right)^{20} - 1\right]}{\frac{106}{100} - 1} = \frac{20000\left[\left(\frac{106}{100}\right)^{20} - 1\right]}{6}$$

To perform this calculation, we must have recourse to logarithms, by which we have $L\left(\frac{106}{100}\right)^{20} = 20\,(L\,106 - L\,100) = 0{,}506120$;* whence we draw,

* To find the number correſponding to this logarithm, which is not found exactly in the tables, we cannot, without incurring a ſenſible error, make uſe of the proportion eſtabliſhed, (page 169) between the differences of the number, and thoſe of their logarithms, becauſe in the firſt ten of the tables, the difference between the logarithms of two conſecutive numbers is conſiderable, and the ſaid proportion can take place only when that difference is very ſmall; whence, it must be concluded, that the greater two conſecutive numbers are, the more exact is the operation, ſince the difference of their logarithms becomes ſmaller.

An eaſy way to apply this remark, is to increaſe the characteriſtic of the logarithm by as many units as the extent of the tables can allow, and then taking the number neareſt to the log. ſeparate with a comma, as many decimals, as units you have added to the characteriſtic of the log.

Here we have been ſatisfied with getting the number within one hundredth, and conſequently we have increaſed the characteriſtic only by two units, which has given 321 for the neareſt number to the log. and then 3,21 for the number demanded.